RC **circuits** (`EC_rc_2.sqproj`)

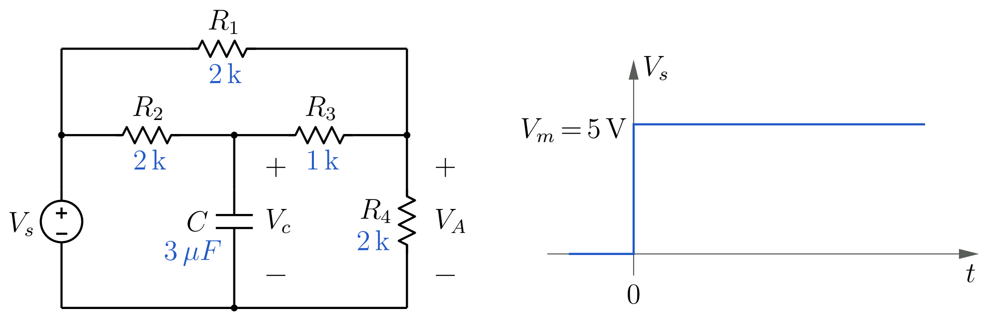

Figure 1: RC circuit with a step input.

Question: For the circuit shown in Fig. 1, find the following.

(a) time constant

(b) $V_c(0^-)$ and $V_A(0^-)$

(c) $V_c(0^+)$ and $V_A(0^+)$

(d) $V_c(\infty)$ and $V_A(\infty)$

Using the above values, plot $V_c(t)$ and $V_A(t)$.

Solution:

(a) The circuit time constant is given by $\tau = R_{Th}C$, where R_{Th} is the Thevenin equivalent resistance seen by the capacitor. To obtain R_{Th}, we deactivate the voltage source and find the resistance seen from the capacitor, as shown in Fig. 2. After simplifying the circuit, we get

$$R_{Th} = R_2 \parallel [R_3 + (R_1 \parallel R_4)]\,. \tag{1}$$

For the component values given in Fig. 1, we get $R_{Th} = 1\,\text{k}$, and $\tau = R_{Th}C = 3\,\text{msec}$. We can expect the transients to vanish after about $5\tau = 15\,\text{msec}$.

(b) Since the source voltage has been $0\,\text{V}$ for a long time before $t = 0$, we have $V_c(0^-) = V_A(0^-) = 0\,\text{V}$.

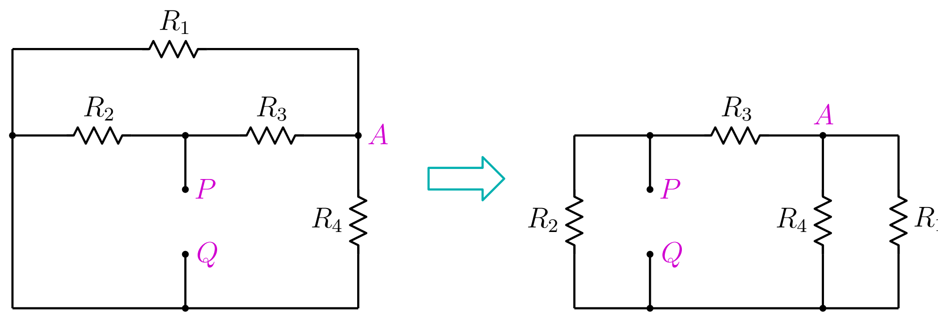

Figure 2: Calculation of the Thevenin resistance seen by the capacitor in the circuit of Fig. 1.

(c) Since the capacitor voltage cannot change abruptly between $t = 0^-$ and $t = 0^+$, we have $V_c(0^+) = V_c(0^-) = 0\,\mathrm{V}$, as shown by a short circuit in Fig. 3. Writing KCL at node A, we get

$$\frac{V_m - V_A}{R_1} = \frac{V_A}{R_3} + \frac{V_A}{R_4} \rightarrow V_A \left[\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \right] = \frac{V_m}{R_1}, \tag{2}$$

where we have taken node Q as the reference node ($0\,\mathrm{V}$). Solving Eq. 2, we get $V_A(0^+) = 1.25\,\mathrm{V}$.

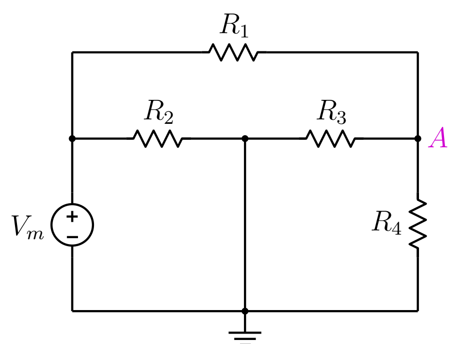

Figure 3: Circuit of Fig. 1 at $t = 0^+$.

(d) As $t \to \infty$, all voltages and currents become constant, the capacitor current $i_c = C\,\dfrac{dV_c}{dt}$ becomes zero, and the capacitor can therefore be replaced with an open circuit, as shown in Fig. 4. Writing KCL at node A, we get

$$\frac{V_m - V_A}{R_1} + \frac{V_m - V_A}{R_2 + R_3} = \frac{V_A}{R_4} \rightarrow V_A \left[\frac{1}{R_1} + \frac{1}{R_2 + R_3} + \frac{1}{R_4} \right] = V_m \left[\frac{1}{R_1} + \frac{1}{R_2 + R_3} \right]. \tag{3}$$

Solving this equation, we get $V_A(\infty) = 3.125\,\mathrm{V}$. $V_c(\infty)$ can now be obtained as

$$V_c(\infty) = V_m \frac{R_3}{R_2 + R_3} + V_A(\infty) \frac{R_2}{R_2 + R_3}. \tag{4}$$

Substituting numerical values, we get $V_c(\infty) = 3.75\,\mathrm{V}$.

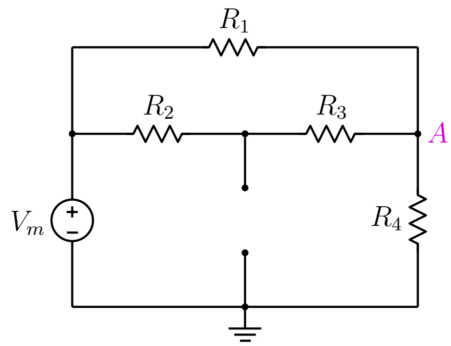

Figure 4: Circuit of Fig. 1 as $t \to \infty$.

Using the above results, we can now plot $V_c(t)$ and $V_A(t)$ (see Fig. 5).

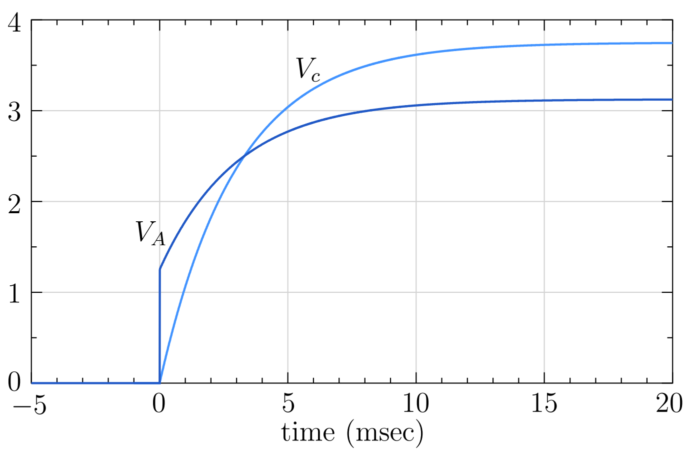

Figure 5: $V_c(t)$, $V_A(t)$ for the circuit of Fig. 1.

SequelApp Exercises: Repeat the above steps for the following situations, with other parameters the same as in Fig. 1. Verify your answers using SequelApp.

1. $R_3 = 3\,\mathrm{k}$.

2. $R_2 = 1\,\mathrm{k}$.