RC **circuits** (`EC_rc_3.sqproj`)

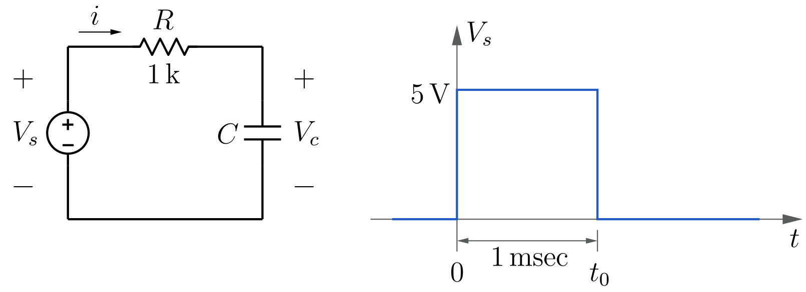

Figure 1: RC circuit example.

Question: In the circuit shown in Fig. 1, V_c is initially $0\,\text{V}$ (i.e., before the input pulse is applied). The capacitance C is $0.5\,\mu\text{F}$.

(a) Sketch $V_c(t)$ and $i(t)$.

(b) Find the following values: (i) $V_c(t_0^-)$, (ii) $V_c(t_0^+)$, (iii) $i(t_0^-)$, (iv) $i(t_0^+)$.

Solution:

The waveforms $V_c(t)$ and $i(t)$ are shown in Fig. 2. The falling edge of the input voltage at $t = t_0$ does not affect the circuit response for $0 < t_0 < t$. In other words, we can compute the response for a step change in V_s (see the dashed part in Fig. 2) rather than a pulse, and simply take the solution for $0 < t_0 < t$.

For this condition (i.e., a step change in V_s), let $V_c(t) = A_1 e^{-t/\tau} + B_1$. Since $V_c(t)$ is the capacitor voltage, it does not change instantaneously, which means $V_c(0^+) = V_c(0^-) = 0\,\text{V}$.

As $t \to \infty$, $V_c \to 5\,\text{V}$ since the capacitor current $i = \dfrac{dV_c}{dt}$ is zero in the steady state, leading to zero voltage drop across R. Using the above two conditions, we obtain $A_1 = -5$, $B_1 = 5$ (volts).

At $t = t_0^-$, $V_c(t)$ can now be evaluated as

$V_c(t_0^-) = (-5)e^{-t/RC} + 5 = (-5)e^{-1\,\text{msec}/0.5\,\text{msec}} + 5 = 4.32\,\text{V}$.

$V_c(t_0^+)$ is the same as $V_c(t_0^-)$ since the capacitor voltage does not change instantaneously.

The current transient for $0 < t_0 < t$ can be obtained in two ways: (i) by differentiating $V_c(t)$, since $i = C\,\dfrac{dV_c}{dt}$, (ii) by computing $i(t)$ for a step change in V_s to obtain $i(t)$ for $0 < t_0 < t$.

Let us use the second approach, with $i(t) = A_2 e^{-t/\tau} + B_2$. At $t = 0^+$, we have $V_s(0^+) = 5$ and $V_c(0^+) = 0$. The voltage drop across R is therefore $5\,\text{V}$, which means $i(0^+) = \dfrac{5\,\text{V}}{R} = 5\,\text{mA}$. As $t \to \infty$, $i \to 0$. These two conditions give $A_2 = 5\,\text{mA}$, $B_2 = 0$, resulting in $i(t) = (5\,\text{mA})\,e^{-t/\tau}$.

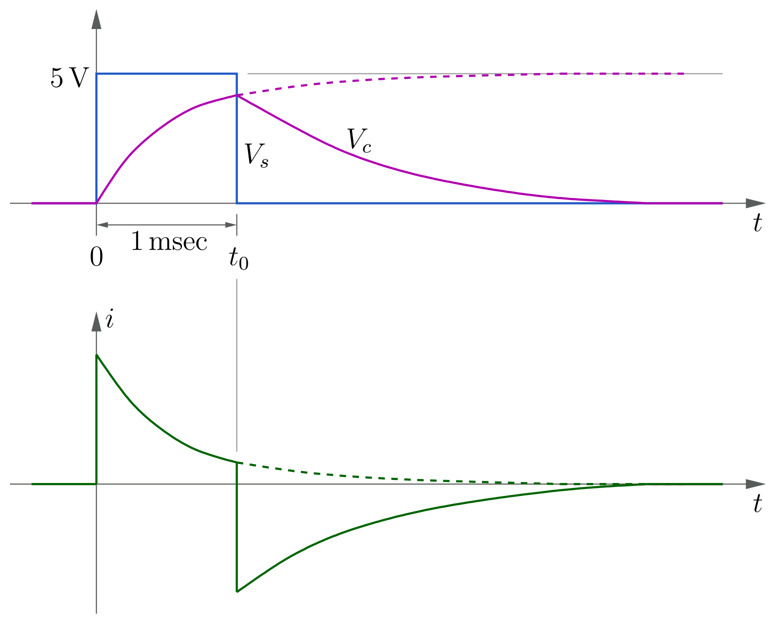

Figure 2: Schematic diagram showing $V_c(t)$ and $i(t)$ for the circuit of Fig. 1.

At $t = t_0^-$, the above expression gives $i(t) = 0.68\,\text{mA}$. The current at t_0^+ can be obtained as $i(t_0^+) = \dfrac{V_s(t_0^+) - V_c(t_0^+)}{R} = \dfrac{0 - 4.32}{R} = -4.32\,\text{mA}$.

SequelApp Exercises: Find C for each of the following conditions:

1. $V_c(t_0^-) = 3\,\text{V}$.

2. $i(t_0^-) = 1\,\text{mA}$.

3. $i(t_0^+) = -1\,\text{mA}$.

Verify your answers using SequelApp.