RL **circuits** (`EC_rl_1.sqproj`)

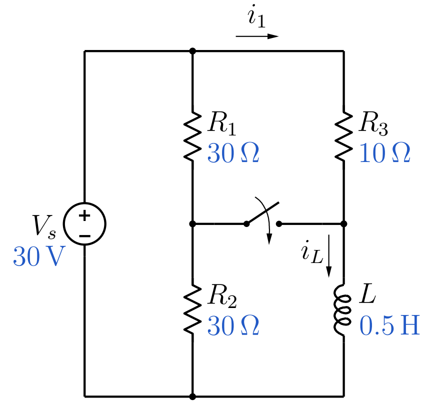

Figure 1: RL circuit example.

Question: In the circuit shown in Fig. 1, the switch has been open for a long time and closes at $t=0$. Plot $i_L(t)$ and $i_1(t)$.

Solution:

First, let us look at the circuit at $t=0^-$ (see Fig. 2 (a)). Since the switch has been closed for a long time, the circuit is in steady state, i.e., the currents and voltages are constant. Since $v_L = L\,\dfrac{di_L}{dt}$ for an inductor, $v_L(0^-)$ must be zero (since $\dfrac{di_L}{dt}=0$) which means that the inductor behaves like a short circuit, as shown in Fig. 2 (b), and we find that $i_1(0^-) = i_L(0^-) = \dfrac{V_s}{R_3} = \dfrac{30\,\text{V}}{10\,\Omega} = 3\,\text{A}$.

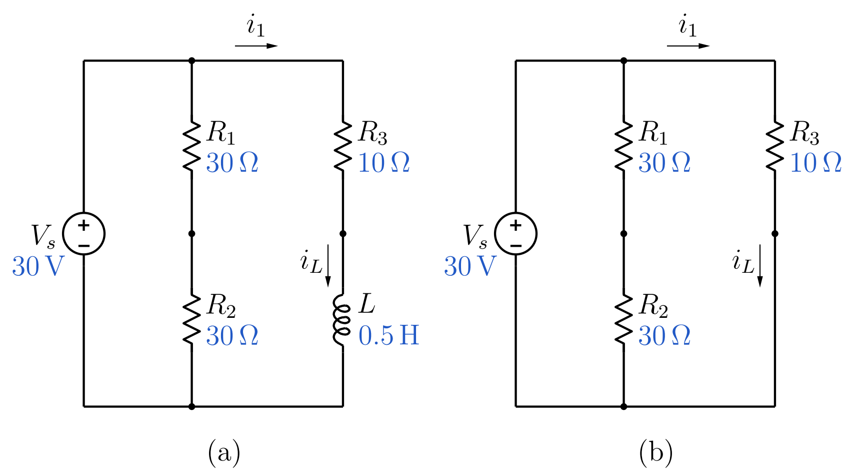

Figure 2: (a) Circuit of Fig. 1 at $t=0^-$, (b) circuit after replacing the inductor with a short circuit.

The circuit after the switch closes at $t=0$ is shown in Fig. 3 (a). The current $i_L(0^+)$ must be

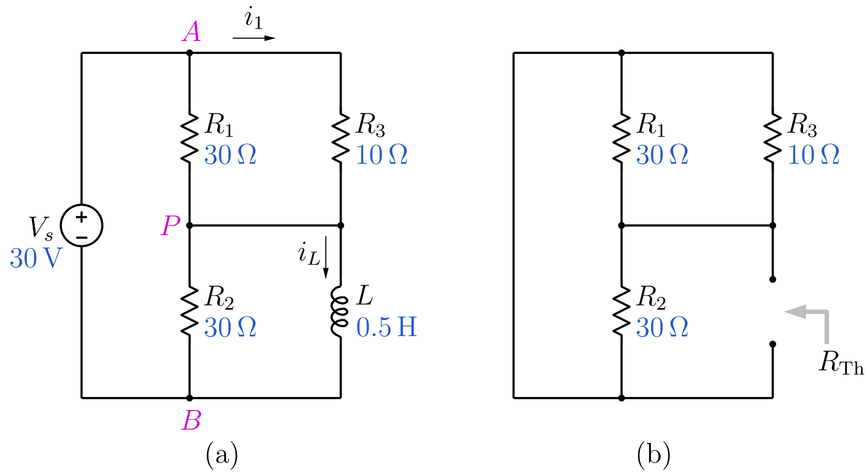

Figure 3: (a) Circuit of Fig. 1 for $t > 0$, (b) calculation of the Thevenin resistance seen by the inductor.

the same as $i_L(0^-)$; otherwise, the inductor voltage $L\,\dfrac{di_L}{dt}$ would become infinitely large, thus violating circuit equations. To obtain $i_1(0^+)$, we can first obtain the node voltage V_P (see Fig. 3 (a)), taking node B as the reference node (i.e., $V_B = 0$). KCL at node P can be written as

$$\frac{V_P - 0}{R_2} - \frac{V_A - V_P}{R_1} - \frac{V_A - V_P}{R_3} + i_L(0^+) = 0, \tag{1}$$

which gives $V_P = 6\,\mathrm{V}$, and $i_1(0^+) = \dfrac{V_A - V_P}{R_3} = 2.4\,\mathrm{A}$.

To find the circuit time constant, we find the Thevenin resistance as seen from the inductor (see Fig. 3 (b)). For this purpose, we deactivate the voltage source (replace it with a short circuit), and find R_{Th} to be $(R_1 \parallel R_2 \parallel R_3) = 6\,\Omega$. The circuit time constant is $\tau = 0.5\,\mathrm{H}/6\,\Omega = 83.3\,\mathrm{msec}$. After the switch closes at $t = 0$, we expect all transients to vanish in about $5\,\tau$ or $0.42\,\mathrm{sec}$, and all currents and voltages would remain constant thereafter.

Next, we need to obtain $i_1(\infty)$ (i.e., the steady-state value of i_1). The indcuctor can be replaced with a short circuit in the steady state (see Fig. 4), and we get

$$i_L(\infty) = \frac{V_s}{R_1 \parallel R_3} = 4\,\mathrm{A}, \tag{2}$$

$$i_1(\infty) = i_L(\infty) \times \frac{R_1}{R_1 + R_3} = 4\,\mathrm{A} \times \frac{30}{40} = 3\,\mathrm{A}. \tag{3}$$

Putting together the above observations, we obtain the plots for $i_L(t)$ and $i_1(t)$ as shown in Fig. 5.

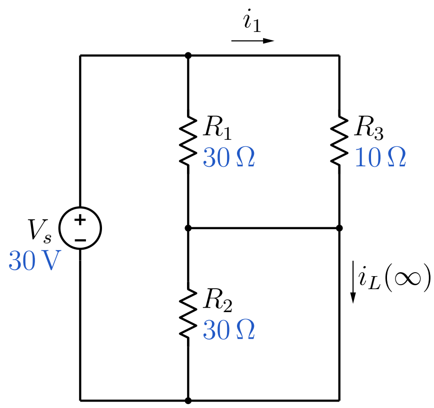

Figure 4: Calculation of $i_L(\infty)$ for the circuit of Fig. 1.

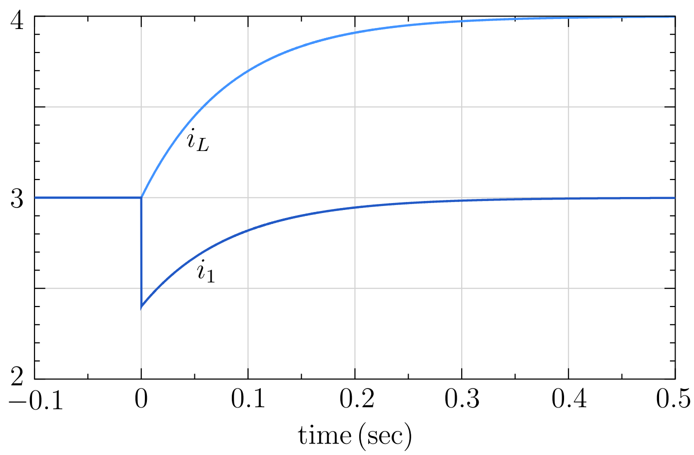

Figure 5: i_L and i_1 versus time for the circuit of Fig. 1.

SequelApp Exercises: Repeat the above steps for the following situations, with other parameters the same as in Fig. 1. Verify your answers using SequelApp.

1. The switch opens (instead of closing) at $t = 0$.

   (The above condition can be simulated by making `val1` $= 1$ and `val2` $= 0$.)

2. R_2 is changed to $10\,\Omega$, R_3 is changed to $30\,\Omega$, and the switch closes at $t = 0$.

3. R_2 is changed to $10\,\Omega$, R_3 is changed to $30\,\Omega$, and the switch opens at $t = 0$.