RL **circuits** (`EC_rl_2.sqproj`)

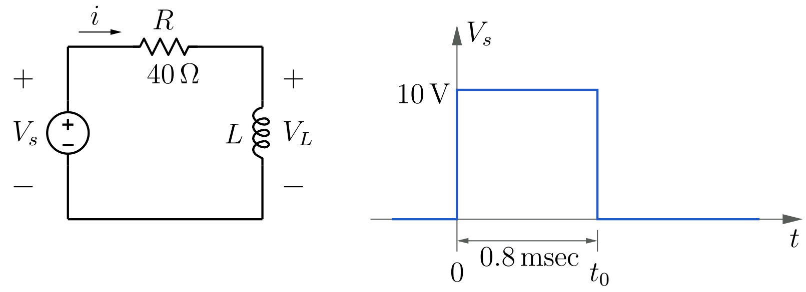

Figure 1: RL circuit example.

Question: In the circuit shown in Fig. 1, i is initially 0 A (i.e., before the input pulse is applied). The inductance L is 20 mH.

(a) Sketch $i(t)$ and $V_L(t)$.

(b) Find the following values: (i) $i(t_0^-)$, (ii) $i(t_0^+)$, (iii) $V_L(t_0^-)$, (iv) $V_L(t_0^+)$.

Solution:

The waveforms $i(t)$ and $V_L(t)$ are shown in Fig. 2. The falling edge of the input voltage at $t = t_0$ does not affect the circuit response for $0 < t_0 < t$. In other words, we can compute the response for a step change in V_s (see the dashed part in Fig. 2) rather than a pulse, and simply take the solution for $0 < t_0 < t$.

For this condition (i.e., a step change in V_s), let $i(t) = A_1 e^{-t/\tau} + B_1$. Since $i(t)$ is the inductor current, it does not change instantaneously, which means $i(0^+) = i(0^-) = 0\,\text{A}$.

As $t \to \infty$, $i \to \dfrac{10\,\text{V}}{R} = 0.25\,\text{A}$ since the inductor voltage $V_L = \dfrac{di}{dt}$ is zero in the steady state.

Using the above two conditions, we obtain $A_1 = -0.25$, $B_1 = 0.25$ (amps).

At $t = t_0^-$, $i(t)$ can now be evaluated as

$i(t_0^-) = (-0.25)e^{-t/RC} + 0.25 = (-0.25)e^{-0.8\,\text{msec}/0.5\,\text{msec}} + 0.25 = 0.2\,\text{A}.$

$i(t_0^+)$ is the same as $i(t_0^-)$ since the inductor current does not change instantaneously.

The V_L transient for $0 < t_0 < t$ can be obtained in two ways: (i) by differentiating $i(t)$, since $V_L = L\,\dfrac{di}{dt}$, (ii) by computing $V_L(t)$ for a step change in V_s to obtain $V_L(t)$ for $0 < t_0 < t$.

Let us use the second approach, with $V_L(t) = A_2 e^{-t/\tau} + B_2$. At $t = 0^+$, we have $V_s(0^+) = 10\,\text{V}$ and $i(0^+) = 0\,\text{A}$, giving $V_L(0^+) = 10\,\text{V}$. As $t \to \infty$, $V_L \to 0$. These two conditions give $A_2 = 10\,\text{V}$, $B_2 = 0$, resulting in $V_L(t) = (10\,\text{V})\,e^{-t/\tau}$.

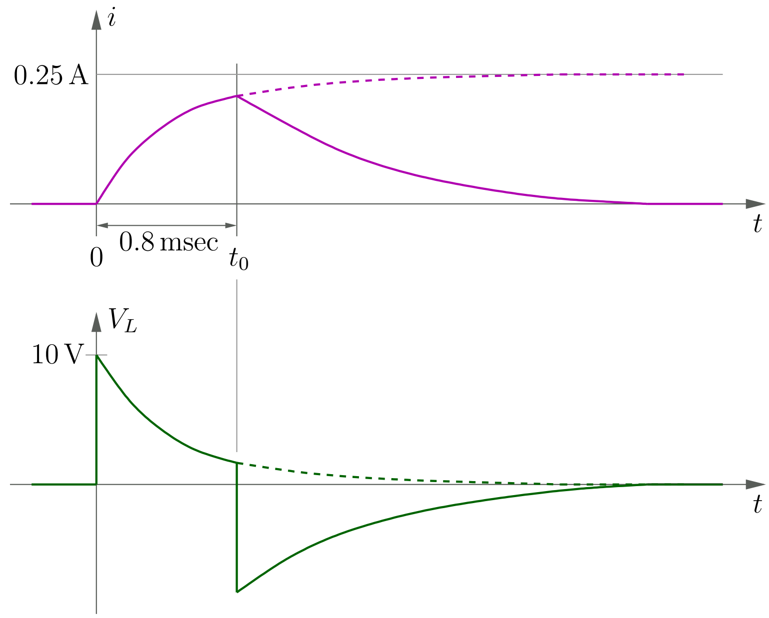

Figure 2: Schematic diagram showing $i(t)$ and $V_L(t)$ for the circuit of Fig. 1.

At $t = t_0^-$, the above expression gives $V_L(t) = 2\,\text{V}$. $V_L(t_0^+)$ can be obtained as
$V_L(t_0^+) = V_s(t_0^+) - i(t_0^+)\,R = 0 - 0.2 \times 40 = -8\,\text{V}$.

SequelApp Exercises: Find L for each of the following conditions:

1. $i(t_0^-) = 0.1\,\text{A}$.

2. $V_L(t_0^-) = 5\,\text{V}$.

3. $V_L(t_0^+) = -6\,\text{V}$.

Verify your answers using SequelApp.