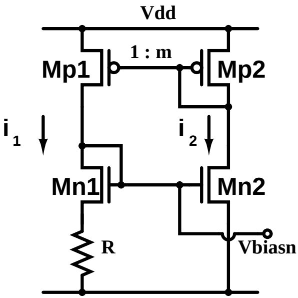
The Beta Multiplier



We have $I_2 = mI_1$ with $m = \frac{(W/L)_{p2}}{(W/L)_{p1}}$
Equating voltages at the gates of n transistors:

$$V_{GS1} + RI_1 = V_{GS2}$$

$$V_{Tn} + \sqrt{\frac{2I_1}{K_{n1}}} + RI_1 = V_{Tn} + \sqrt{\frac{2I_2}{K_{n2}}}$$

$$\text{so } \sqrt{\frac{2I_1}{K_{n1}}} + RI_1 = \sqrt{\frac{2mI_1}{K_{n2}}}$$

Therefore,

$$RI_1 = \sqrt{\frac{2I_1}{K_{n1}}}\left(\sqrt{\frac{mK_{n1}}{K_{n2}}} - 1\right)$$

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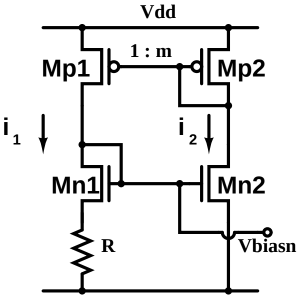

$$RI_1 = \sqrt{\frac{2I_1}{K_{n1}}}\left(\sqrt{\frac{mK_{n1}}{K_{n2}}} - 1\right)$$

$$\text{so } \sqrt{I_1} = \frac{1}{R}\sqrt{\frac{2}{K_{n1}}}\left(\sqrt{\frac{mK_{n1}}{K_{n2}}} - 1\right)$$

$$\text{or } \sqrt{2K_{n1}I_1} = \frac{2}{R}\left(\sqrt{\frac{mK_{n1}}{K_{n2}}} - 1\right)$$

But $\sqrt{2K_{n1}I_1} = g_{mn1}$, $\quad m = \dfrac{(W/L)_{p2}}{(W/L)_{p1}}$

$$\text{Therefore } g_{mn1} = \frac{2}{R}\left(\sqrt{\frac{(W/L)_{p2}/(W/L)_{p1}}{(W/L)_{n2}/(W/L)_{11}}} - 1\right) = \text{ constant}$$

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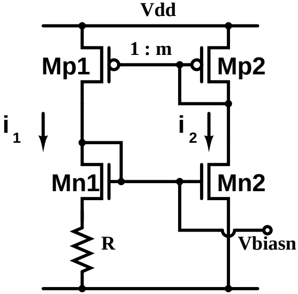


$$g_{mn1} = \frac{2}{R}\left(\sqrt{\frac{(W/L)_{p2}/(W/L)_{p1}}{(W/L)_{n2}/(W/L)_{11}}} - 1\right)$$

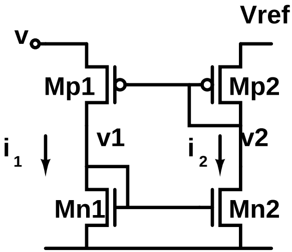
Thus g_{mn1} depends only on R and geometry.

Also, *any* nMOS transistor on the same chip whose current is proportional to I_1 or I_2 will have a g_{mn} which is unaffected by temperature, supply variation etc.

Thus the gate voltage of the n channel transistors (V_{biasn}) can be used to bias n channel transistors on the chip, whose g_m is to be kept constant.

Zero input impedance circuit

Low r_{in} amps are used for photo-detectors. (C.-K. Kim et al, "High Injection Efficiency Readout Circuit for Low Resistance Infrared Detector", IEE Electronic Letters, 35, 1507, 1999).



$$\begin{aligned} i_1 &= g_{mn1} v_1 &&= g_{mp1}(v - v_2) \\ i_2 &= g_{mn2} v_1 &&= -g_{mp2} v_2 \\ v_2 &= -\frac{g_{mn2}}{g_{mp2}} v_1 &&= -\frac{g_{mn2}}{g_{mp2}} \frac{i_1}{g_{mn1}} \end{aligned}$$

$$i_1 = g_{mp1} v + \frac{g_{mn2}/g_{mn1}}{g_{mp2}/g_{mp1}} i_1$$

define $\Gamma \equiv \dfrac{g_{mn2}/g_{mn1}}{g_{mp2}/g_{mp1}}$ then, $i_1(1-\Gamma) = g_{mp1} v$

This gives $r_{in} = (1-\Gamma)/g_{mp1}$

By making Γ = 1, we can make $r_{in} = 0$.

Robustness of design

In saturation,

$$I_d = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_g - V_T)^2$$

$$\text{So, } g_m = \mu C_{ox}\frac{W}{L}(V_g - V_T) = \sqrt{2\mu C_{ox}\frac{W}{L}I_d}$$

$$g_{mn2}/g_{mn1} = \sqrt{\frac{(W/L)_{n2}}{(W/L)_{n1}}\frac{I_2}{I_1}}$$

$$g_{mp2}/g_{mp1} = \sqrt{\frac{(W/L)_{p2}}{(W/L)_{p1}}\frac{I_2}{I_1}}$$

$$\text{Therefore } \Gamma \equiv \frac{g_{mn2}/g_{mn1}}{g_{mp2}/g_{mp1}} = \sqrt{\frac{(W/L)_{n2}/(W/L)_{n1}}{(W/L)_{p2}/(W/L)_{p1}}}$$

Thus Γ depends only on geometry and is independent of temperature and device parameter variation.