INVESTIGATIONS ON

WAVELET-BASED ECG DENOISING

A dissertation submitted in

partial fulfillment of the requirements for the degree of

Master of Technology

in Biomedical Engineering

by

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July 2014

Indian Institute of Technology Bombay

M. Tech. Dissertation Approval

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Dhaval G. Shah / Prof. P. C. Pandey (Supervisor), "Investigations on wavelet-based ECG denoising", M.Tech. Dissertation, Biomedical Engineering, Indian Institute of Technology Bombay, July 2014.

ABSTRACT

Ambulatory ECG recording are often corrupted by baseline wander (BW), EMG noise, and motion artifact (MA). These artifacts make it difficult to measure the duration and amplitude of P wave, time interval between characteristic points, dip or elevation of ST segments from isoelectric point. For suppression of these artifacts, an investigations on a wavelet-based denoising technique is carried out.

Objective evaluation of denoising technique is carried out by applying it on ECG with simulated noise, obtained by adding noise-free ECG and ECG-free noise, and calculating SNR improvement and correlation coefficient. Use of correlation coefficient is extended to decompose the error in the output with respect to the noise-free reference to get the estimates of signal attenuation, noise attenuation, and magnitude of distortion for devising further improvements. An automated method for calculating insertion and detection errors in R-peak detection as a function of temporal tolerance is developed for assessing the usefulness of the denoising technique for arrhythmia detection.

A wavelet-based denoising technique developed using discrete Meyer wavelet, smooth thresholding and smooth limiting of wavelet coefficients, and thresholds determined from the signal statistics and two externally supplied control parameters is investigated for suppression of BW, EMG noise, and MA in ECG and to study the effect of the control parameters. Results of subjective and objective evaluation showed the effectiveness of the denoising technique. The control parameters selected for usefulness over a range of input SNR values resulted in SNR improvement of 18.5, 6, 8.3 dB for BW, EMG noise, and MA, respectively for input SNR of – 12 dB. Results showed that the denoising can be made more effective by selecting the denoising control parameters on the basis of an assessment of the level and type of artifacts.

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List of Abbreviations

BIH	Beth Israel Hospital, Boston, Massachusetts
BW	baseline wander
CDF	cumulative distribution function
DWT	discrete wavelet transform
ECG	electrocardiogram
EMD	empirical mode decomposition
EMG	electromyogram
FIR	finite impulse response
ICA	independent component analysis
II	improvement index
IMF	intrinsic mode function
LMS	least mean squares
MA	motion artifact
MIT	Massachusetts Institute of Technology
MSE	mean square error
RMSE	root-mean-square error
PRD	percentage root-mean-square difference
SNR	signal-to-noise ratio
SWT	stationary wavelet transform
TIWT	translation-invariant wavelet transform

List of Symbols

$d_j(i)$	detail at scale *j*
$D_j(i)$	unmodified wavelet coefficients at scale *j*
$\hat{D}_j(i)$	unmodified wavelet coefficients at scale *j*
$e(n)$	error signal
r	correlation coefficient
S_i	transition span
$s(n)$	noise-free signal
$x(n)$	noisy signal
$y(n)$	processed output signal
ε	EMG denoising control parameter
$\phi_i(n)$	limiting threshold for the i^{th} decomposition level
$\nu(n)$	time-varying thresholding factor
β	scaling coefficient for the input signal
γ	attenuation coefficient for the input noise
κ	distortion coefficient
η	MA denoising control parameter
μ	Mean
$\theta_i(n)$	time-varying threshold for the i^{th} scale
σ	standard deviation

Chapter 1
INTRODUCTION

1.1 Problem overview

Electrocardiogram (ECG) [1]-[3] is a bio-signal associated with electrical activity of the heart muscles. It is picked-up using surface electrodes and is used for diagnosing cardiac disorders. Artifacts in the picked-up signal may distort its important features and may lead to false detection or false rejection of a disease. Modern hardware used to acquire ECG under clinical conditions with the patient under rest is generally capable of artifact-free recording. However, many physiological disorders can be diagnosed only from the recordings during stress test or from the extended-duration recordings made with the patient carrying out regular activities of daily life. Recording made with patient performing normal activities like sitting, climbing stairs, etc is known as ambulatory recording and is usually carried out using a wearable Holter monitor. Recordings of stress test ECG and ambulatory ECG are often corrupted by several artifacts like baseline wander (BW), electromyogram (EMG), and motion artifacts (MA). BW refers to a deviation in the baseline caused by respiration or movement of the patient. Higher spectrum of baseline wander may have some overlap with the lower spectrum of ECG. EMG artifact is caused by involuntary or voluntary muscular contraction. Lower part of its spectrum overlaps with the higher part of ECG spectrum. Good electrode-skin contact is required for acquiring ECG signal and this is generally achieved by applying an electrolyte gel between the electrode and skin. Differences in the motion of the two electrodes involved in sensing an ECG signal creates imbalance in the electrical activities at the electrode-electrolyte and electrolyte-skin interfaces at the two electrode sites and contributes a time-varying potential component in the sensed signal. This motion related component is known as the motion artifact. MA can be minimized by controlling the motion of the limbs, but this is not practical in ambulatory recordings [4]. This artifact is difficult to eliminate because of its spectral overlap with ECG. ECG recordings may have some white Gaussian noise or pink noise (predominately low frequency noise), generated by the electronic circuits used for signal conditioning and acquisition. The recordings may also be contaminated by external interferences, mainly the power line interference and the interference caused by nearby medical or other electronic devices.

The artifacts in ECG recordings may mask some of the diagnostically important features, like the ST segment, and thus may reduce the diagnostic usefulness of the signal [5]. Hence there is a need for suppressing these artifacts from the ECG recordings. ECG spectrum extends over 0.05−150 Hz, while the BW, MA, and EMG have spectra extending over 0.01−1 Hz, 1−10 Hz, and 5−500 Hz, respectively. As the artifacts have spectra with significant overlap with ECG spectrum, artifacts cannot be eliminated by time-invariant filtering and advanced signal processing techniques are required for the denoising.

1.2 Project objective

The objective of the project is to develop and evaluate denoising techniques to suppress the artifacts in ECG by using wavelet-based denoising methods. BW, EMG, and MA which affect the stress test and ambulatory recordings of ECG are to be suppressed to improve the usefulness of these recordings. The project consists two investigations. The first investigation involves detailed assessment of the subjective and objective techniques used for the evaluation of the denoising techniques and proposal of a few supplementary techniques. The second investigation involves use of denoising based on thresholding and limiting of wavelet coefficients and its evaluation.

1.3 Dissertation outline

Chapter 2 presents a review of the artifacts suppression techniques. Chapter 3 gives a description of ECG databases used for the investigation, a review of subjective and objective methods for evaluating the denoising techniques, and description of a few proposed supplementary objective methods. The fourth chapter gives a description of the wavelet based denoising method using thresholding and limiting of wavelet coefficients. Chapter 5 presents the subjective and objective evaluation of the denoising technique described in the forth chapter Chapter 6 provides summary and conclusions along with some suggestions for future work.

Chapter 2
DENOISING TECHNIQUES

2.1 Introduction

ECG recordings may be contaminated by artifacts like baseline wander (BW), EMG noise, and motion artifact (MA). Figure 2.1 shows the relative power spectra of ECG, QRS complex, P-T waves, MA, and EMG noise [2]. Because of a significant overlap between the ECG and artifact spectra, linear time-invariant filtering is not very effective in suppressing the artifacts. In the presence of artifacts in ECG, it becomes difficult to measure the duration and amplitude of P wave, time interval between characteristic points, dip or elevation of ST segments from isoelectric point [3]. For suppressing the artifacts, several techniques, like digital filtering [6], [7], adaptive filtering [3], [8], [9], independent component analysis [5]-[10], empirical mode decomposition [11], [12], and discrete wavelet transform [4], [13]-[19] have been reported. This chapter provides a review of some of these techniques.

2. 2 Digital filter based denoising

Alste and Schilder [6] used a digital filter to suppress the powerline interference and BW. A digital nonrecursive band-pass FIR filter was designed with the emphasis on reducing the number of coefficients. Filters were designed for reducing powerline interference and BW, with a lower cutoff frequency 0.8 Hz. It may be noted that the American Heart Association recommendation is that the lower cutoff for ECG should not be greater than 0.05 Hz, possibly to avoid phase distortion introduced by analog or recursive digital filters. Tests performed on ECG sampled at 250 Hz and corrupted by BW, EMG artifact, and power-line interference showed satisfactory results in almost all cases. The test showed removal of BW during exercise except when the BW period was near to that of the heart beat.

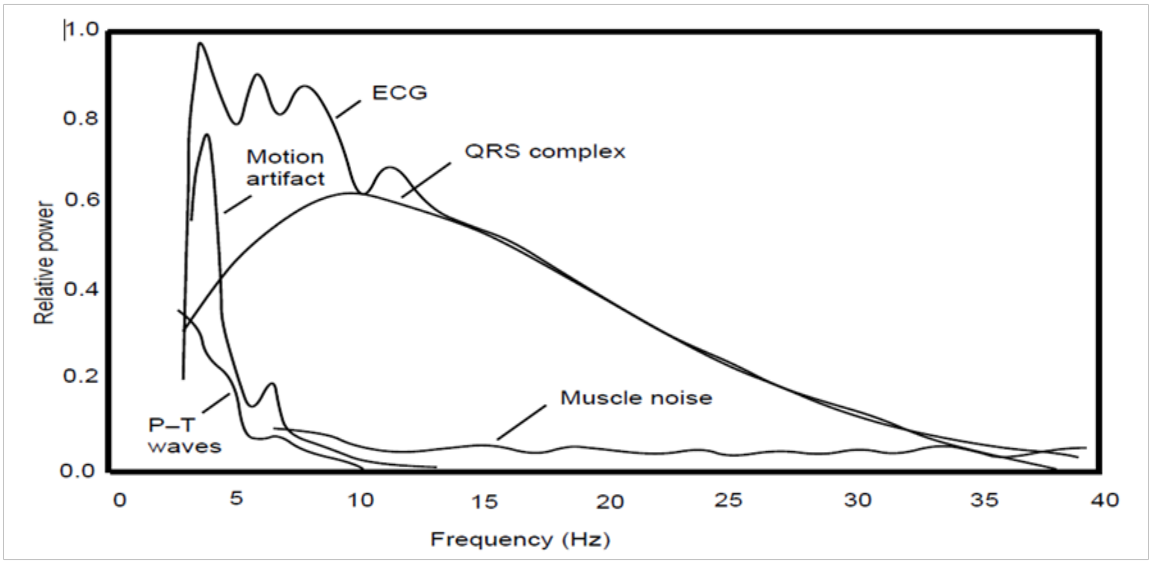


Figure. 2.1: Relative power spectra of ECG, QRS complex, P and T waves, muscle noise and motion artifacts based on an average of 150 beats [2].

Lee and Bien [7] devised a variable bandwidth filter to suppress the powerline interference and EMG artifact from the ECG signal. As the overlaps between the spectrum of ECG and that of the noise continuously change, a variable bandwidth filter was designed to suppress the noise outside the instantaneous bandwidth. It was tested on noisy ECG signal obtained from a portable 24-hour health monitoring system. The lower cut-off frequency was 1 rad/sec and the upper cut-off frequency was 430 rad/sec. The SNR improvement was reported to be 11 dB greater than that achieved by a linear time-invariant filter.

2.3. Adaptive filter based denoising

To remove various artifacts in ECG, Rahman et al [3] proposed normalized signed regressor LMS as a simple and efficient technique. Computational complexity of the adaptive algorithm was reduced without affecting the output signal quality. It was tested on MIT-BIH arrhythmia database and artifacts from noise stress database. Results regarding BW, powerline interference, muscle artifacts, and MA showed substantial improvement.

Tong et al [8] used the hypothesis that MA in ECG can be reduced by using motion of electrode as the reference. Motion sensing was carried out using (i) two-axis anisotropic magneto-resistive sensor and (ii) three-axis accelerometer developed using two dual-axis accelerometers. Adaptive filter shown in Figure 2.2 was implemented in Matlab. Lowpass filter was used to decrease the amplitude of QRS complex. LMS algorithm was implemented for determining

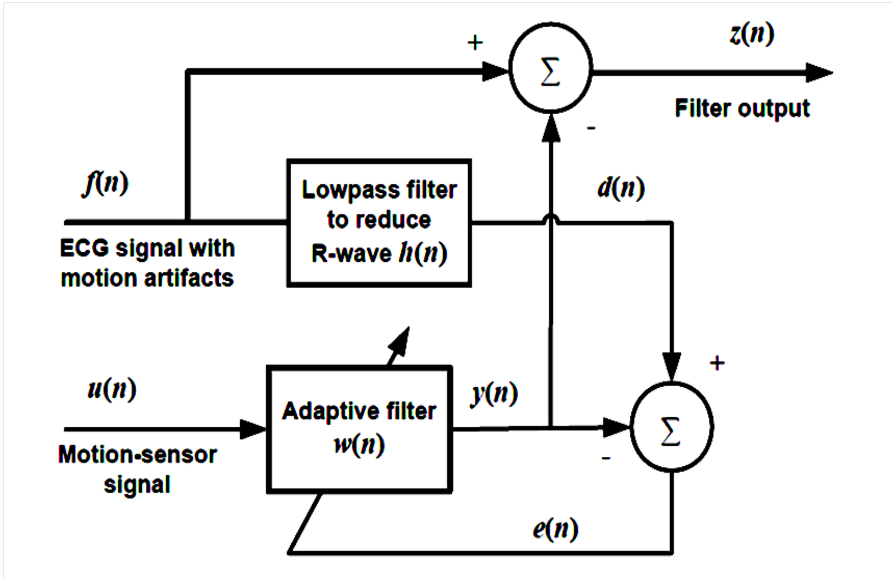


Figure 2.2: Block diagram of adaptive filter [8]

adaptive filter weight. MA was stimulated by combination of the following conditions: (i) pushing electrode, (ii) pushing skin around electrode, and (iii) pulling electrode wires. Testing was carried out on recordings from 8 subjects and under five conditions for introducing different types of artifacts. In addition to the three conditions mentioned above; fourth condition was with combination of all three kinds of artifacts and fifth condition was artifact-free ECG. It was observed that using the electrode motion as input to an adaptive filter reduced MA, and use of accelerometer performed better than use of magneto-resistive sensor.

Wu et al [9] used normalized adaptive neural filter (NANF) for suppressing the muscle artifact and the high-frequency noise in ECG signals. Steepest-descent algorithm was used for updating normalized filter coefficients. To smooth the ECG waveshape and to suppress 60 Hz power-line interference, 12^{th} order Butterworth lowpass filter was combined with moving-average (MA) lowpass filter. Function of NANF was to modify the coefficients based on reference input. For evaluation, MIT-BIH arrhythmia database was used along with the RMS error, normalized correlation co-efficient, SNR, and filtered-artifact entropy as performance indices. The test results showed that the technique gave good improvement in all the indices as compared with LMS filtering and that it retained the waveshape of the ECG signal.

2. 4 Denoising based on independent component analysis (ICA)

ICA is based on the assumption that the signals from *n* different sources are mixed together and we have *n* such mixtures available. This technique separates the sources in a blind manner, with the objective of estimating the original shapes although the amplitudes may not get correctly estimated. It has been proposed that ICA can be used for suppressing the artifacts even if signal and artifact have spectral overlap. In [5], ICA algorithm with self-adaptive step-size was used for suppression of artifacts in biosignals, like ECG, to accelerate the speed of convergence [5]. Test performed on ECG records from the "nstdb" records of MIT-BIH database showed satisfactory results, with two-layer network of ICA algorithm found to be more effective. Foresta et al [10] reported a technique called WICA, combining the characteristics and advantages of wavelet transform and ICA. Wavelet transforms was used for filtering and also as the preprocessing step for signal decomposition using an *n*-dimensional orthogonal basis and then ICA technique was applied for suppression of artifacts. Evaluation carried on real ECG signals mixed with real artifacts showed a correlation of about 0.9 between original and output waveforms.

2.5 Denoising based on empirical mode decomposition

Empirical mode decomposition (EMD) does not require any predefined basis function to represent a signal, and is considered to be well suited for biomedical signals which are non-stationary in nature. EMD decomposes the signal into a sum of intrinsic mode functions (IMFs) [11]. For a function to become IMF, number of maxima and minima can differ at the most by one and mean of upper and lower envelop should be zero [16]. Using an algorithm called sifting, the IMFs are obtained for the input signal $\mathbf{x}$, by applying the following steps:

i) Set the iterating variable i = 1 and the first proto IMF as $\mathbf{h}_0 = \mathbf{x}$.

ii) Obtain the local maxima and minima of the proto IMF $\mathbf{h}_{i-1}$.

iii) Locate the local maxima of $\mathbf{h}_{i-1}$ and connect them using a cubic spline to get an upper envelope $\mathbf{e}_\mathrm{u}$. Similarly obtain the lower envelope $\mathbf{e}_\mathrm{l}$ by connecting the local minima. Obtain the mean of the $\mathbf{e}_\mathrm{u}$ and $\mathbf{e}_\mathrm{l}$ as

$$\mathbf{m}_i = (\mathbf{e}_u + \mathbf{e}_i)/2 \qquad (2.1)$$

and use it to calculate the *i*th proto IMF as given below:

$$\mathbf{h}_i = \mathbf{x} - \mathbf{m}_i \qquad (2.2)$$

iv) Calculate the stopping criterion. One of the widely used stopping criterion is the normalized sum of difference (SD) [19] between the successive iterations and given as the following:

$$\mathrm{SD}_i = \sum_{k=0}^{N} \frac{\left[h_{i-1}(k) - h_i(k)\right]^2}{h^2{}_{i-1}(k)} \tag{2.3}$$

where N is the length of the signal. If SD is below a predefined threshold, proceed to the next step. Else increment i and go to step (ii).

v) As the stopping criterion is satisfied take the proto IMF h_i as the first IMF $\mathbf{c}_1$. The IMF is subtracted from the input signal to get the residue.

$$\mathbf{r}_1 = \mathbf{x} - \mathbf{c}_i \tag{2.4}$$

As there is good probability of residue containing another IMF, it is analyzed as the input signal to obtain the second IMF and corresponding residue. In these way all possible IMFs can be obtained until the residue obtained is monotonically increasing function, a constant, or a function with only one peak. This way the signal is decomposed into IMFs and may be written as the following:

$$\mathbf{x} = \sum_{i=1}^{J} \mathbf{c}_i + \mathbf{r}_J \tag{2.5}$$

The IMF of different scales represents different frequency bands of the signal which depends on the waveform being analyzed. The main advantage of EMD-based decomposition compared to wavelet-based decomposition is that former does not require a predefined basis function [11], [21], [22].

Chacko and Ari [12] used EMD to denoise the ECG. Assuming that noisy IMFs have relatively flatter spectra, they were identified using spectral flatness measure. Test were carried on ten noise-free ECG signals from the MIT-BIH "mitdb" database and added with Gaussian noise. Output SNRs of 9.9, 14.7, and 18.6 dB were reported for input SNRs of 5, 10, and 15 dB, respectively.

2.6 Denoising based on discrete wavelet transform analysis

Zhang [13] used DWT for removal of BW and high frequency noise. BW was estimated by coarse or approximation coefficients of DWT. Position-dependent and level-dependent thresholding values based on empirical Bayes posterior median wavelet shrinkage method were used to reduce the high frequency noise. Two methods for selecting the scale for BW correction were used: (i) visual selection of the detail for matching BW in the signal, (ii) association of the wavelet and scaling functions with half band high-pass and half band low-pass filters, respectively. In both methods, A8 was found to be a good estimate of BW, for signal sampled at 360 Hz. For denoising, sparsity property of wavelet shrinkage was used for preserving sharp features. It was observed that the thresholds had to be carefully selected. Denoising technique was implemented using 6-level decomposition, symmlet-8, level-dependent thresholding, and translation-invariant wavelet transform. Assessment of the denoising using visual inspection showed satisfactory results.

Kania et al [14] used the wavelet technique in arrhythmia and extrasystols. Selection of the mother wavelet as well as choice of level of decomposition was found to be important and significant noise reduction was observed for symmlet-8 for 5th level of decomposition. Visual inspection showed the method to be far better than ensemble averaging especially in arrhythmia and extrasystols conditions.

Patil and Holambe [15] proposed a wavelet thresholding method "maxminlev", in which the threshold at each scale is selected as the following:

$$\theta_j = \left(\max\left|(D_j(i))\right| + \left|\min(D_j(i))\right|\right)/(2l) \qquad (2.6)$$

Median filter was used to suppress BW, and subsequently 3-level wavelet decomposition was carried out using *bior4.4*. Hard thresholding was applied at each scale and ECG was reconstructed. It was tested with MIT-BIH arrhythmia database record 103 as noise-free signal. With white Gaussian noise and input SNR of 9.7 dB, output SNR was 10.3 dB for the proposed thresholding method and 10.6 dB for "sqtwolog" thresholding. With electrode motion related artifact and input SNR of 4.04 dB, output SNR for was 11.2 dB for the proposed thresholding method and 11.8 dB for "rigresure" thresholding.

Mithun [16] investigated the use of different wavelets for removal of BW and MAs in ECG, using scale-dependent thresholding. With the wavelets like Daubechies 8 (db8) and discrete Meyer, significant amount of noise was removed without introducing noticeable distortion. Evaluation was carried out (*i*) qualitatively by visual examination of the processed output waveform, (*ii*) quantitatively by calculating improvement indices based on signal statistics, and (*iii*) by using it as a pre-processing for automated QRS detection. BW was significantly removed and efficiency of automatic R peak detection was increased. No significant suppression was found in EMG artifact and motion artifact.

In general, wavelet-based denoising can be effective if some of the dilated or scaled version of the mother wavelet or the scaling function resemble significant components of either the ECG or the artifact. BW and MA in ECG do not correspond to shape of any of the commonly used wavelets. However, all the wavelets at some dilation are somewhat similar to ECG signal components. Mithun et al [4], [17] used nonlinear modifications of the wavelet coefficients to suppress the EMG and MA. ECG was improved by removing noise related to EMG noise and

BW noise by using discrete Meyer wavelet. For signals with sampling frequency of 360 Hz, EMG artifacts get represented in first four details. In particular, the coefficients $D_1(i)$ are predominantly contributed by EMG and hence can be removed. the coefficients $D_2(i)$ and $D_3(i)$ contain contribution from ECG as well as EMG and hence thresholding needs to be applied on these coefficients. As hard thresholding introduces distortions and soft thresholding may not sufficiently suppress the artifact, a thresholding function was devised combining features of soft and hard thresholding. The wavelet thresholding based noise suppression is based on the assumption that the contribution of the noise to the wavelet coefficients is generally always present and with low amplitude, while contribution of the signal is in specific time segments and with relatively high amplitude. In ECG corrupted with non-stationary MA, ECG signal is always present and the MA occurs intermittently and generally with high amplitude. Therefore, they applied a limiting function on wavelet coefficients for suppressing MA. Denoising resulted in pseudo-Gibbs oscillations which could be effectively suppressed by using denoising based on translation-invariant wavelet transform (TIWT). The technique was evaluated on noisy records obtained by adding artifact from MIT-BIH noise stress test data to artifact-free ECG from MIT-BIH arrhythmia database, with SNR improvement and R-peak detection as performance indices. For input SNR of -10, -5, 0 dB SNR_{impr}. reported was 12.1, 8.8, and 5.1 dB, respectively. False R-peak detection rate decreased from 14.5 % to 2.2 %. The denoising was also evaluated on ambulatory ECG signal recorded by a Holter monitor and visual examination showed improved performance.

Pranava [18] carried out further investigations using the denoising technique reported in [16] and [4] and with discrete Meyer wavelet for EMG and MA suppression. For suppressing the pseudo-Gibbs oscillations introduced during denoising using DWT, use of TIWT and stationary wavelet transform (SWT) was investigated and both showed improvements over DWT, with TIWT being generally better. SNR improvement, L2 norm and max-min based improvement indices, and R-peak detection rate were used as performance indices for quantitative evaluation. Artifact-free ECG signals from "mitdb" and ECG-free artifacts signal from "nstdb" were used together to generate noisy signals with known SNR. For input of SNR −10 dB, SNR improvement of 14.5, 15.0, 14.7 dB was obtained for DWT, TIWT, and SWT, respectively. Denoising resulted in a significant increase in QRS detection and false detection was reduced to half.

Lin et al [19] reported a wavelet-based denoising method, for suppression of BW and EMG, consisting of three steps: (i) thresholding of wavelet coefficients, (ii) R-peak detection, and (iii) windowing of coefficients around R-peaks for EMG suppression and signal reconstruction.

The technique was applied on signals sampled at 360Hz with 8-level wavelet decomposition. Several combinations of different wavelet functions (Haar, DB-4, DB-6, symlet-5, symlet-8, bi-orthogonal-3.5, coiflete-4), transform types (DWT, SWT), and thresholding methods (soft, hard, improved) were investigated by plotting SNR_{impr} against SNR_{in} for noisy signals generated using ECG and artifacts from MIT-BIH database. In the first stage of the processing, the thresholding was applied on detail coefficients, with thresholds selected as $\theta_j = \sigma\sqrt{2\log(N)}$ with $\sigma = \text{median}\left|(D_j)\right| / 0.6457$, and BW was suppressed by setting approximation coefficients as zero. Use of DWT with symlet-5 and soft thresholding resulted in highest SNR_{impr} for low-SNR signals. In the second stage, R-peaks were located by applying a threshold-based method on the detail coefficients $D_3(i)$ to $D_5(i)$. In the third stage, windowing was applied on the wavelet coefficients with windows centered at detected R-peaks with scale-dependent widths. This stage was meant for EMG suppression and hence no windowing was applied on $D_6(i)$ to $D_8(i)$. Technique was shown to be effective in suppressing BW and EMG. Authors have reported that application of their technique resulted in SNR improvement of 12.9, 11.0, and 9.8 dB for ECG with EMG noise and input SNR of −10, −5, 0 dB, respectively, and the improvements were better than the corresponding values of 12.1, 8.8, and 5.1 dB reported by Mithun et al [4].

2.7 Summary

Methods like digital filtering, adaptive filtering, ICA, EMD, and wavelet based denoising techniques have been reviewed for the suppression of artifact and noise in ECG. Due to spectral overlap between the ECG components and noise, linear filters are not effective in suppressing artifacts in ECG. and distortions may get introduced in important feature of the ECG signal like ST segments, T waves etc, although the R-peak amplitudes are generally not affected. Adaptive filtering technique requires the reference of the noise which might be unavailable. ICA needs multi-channel signals which are not available during ambulatory recordings. EMD has been found to be not very effective at low input SNR. Compared to these methods, wavelet-based denoising techniques have been reported to be much more effective and are particularly suited for ambulatory recordings. As the effectiveness of the denoising is dependent on the choice of thresholds used for thresholding and limiting of wavelet coefficients, a detailed evaluation of the effect of the parameters used in threshold selection is needed.

Chapter 3
EVALUATION OF DENOISING TECHNIQUES

3.1 Introduction

Evaluation of a denoising technique involves assessment of removal of the noise and introduction of any distortions in the signal waveform. The evaluation may be carried out using subjective methods involving visual inspection of the processed output and objective methods involving calculation of performance indices. The objective methods generally need noise-free signal as the reference waveform for calculation of the indices. Different objective methods put different emphasis on specific signal features. Hence, we generally need to use a set of performance indices after examining their relevance with respect to the application of the denoised waveform. The subjective methods give only a qualitative assessment but they can be used in the absence of noise-free reference. This chapter gives a review of some of the earlier reported methods and presents a few new supplementary ones.

The ECG signals commonly used for evaluation of the denoising techniques are briefly described in the next section. The third section gives a review of the subjective methods. The fourth section reviews objective methods based on sample-by-sample comparison of two waveforms. Sections 5 provides a review of methods based on waveform features. In the sixth section, a few supplementary methods are proposed. Investigations on the objective methods are described in the two subsequent sections. The last section provides a summary.

3.2 ECG signals used for evaluation of denoising techniques

Most of the investigations related to ECG processing have used databases available from Physionet [23] as listed in Table 3.1. Most of them became available during 1980-1990, while "twadb" was reported in 2008. The databases labeled as MIT-BIH were developed as a collaborative work between MIT and the Arrhythmia Lab at Boston's Beth Israel Hospital (BIH), while "ST-T" database was developed at CNR Institute of Clinical Physiology, Pisa, Italy.

Table 3.1: ECG databases from Physionet [30], [32]

ECG Database	Sampling Freq. (Hz)	Resolution (bits)
MIT-BIH Normal Sinus Rhythm Database (nsrdb) [24]	128	12
MIT-BIH Arrhythmia Database (mitdb) [25]	360	11
MIT-BIH Noise Stress Test Database (nstdb) [26]	360	11
MIT-BIH Atrial Fibrillation Database (afdb) [27]	250	12
MIT-BIH ST Change Database (stdb) [28]	360	12
MIT-BIH Malignant Ventricular Ectopy Database (vfdb) [29]	250	12
T-Wave Alternans Challenge Database (twadb)[30]	500	16
Sudden Cardiac Death Holter Database (sddb) [31]	250	12

The ECG recordings in "MIT-BIH normal sinus rhythm database" or "nsrdb" [24], [32] contain signals with negligible arrhythmia, recorded from 18 subjects (5 men with age of 26-45 years, 13 women with age of 20-50 years). The "MIT-BIH arrhythmia database" or "mitdb" [25], [32] contains 48 half-hour excerpts from 24-hour 2-channel ambulatory ECG recordings, from 47 subjects with two recordings from one subject and one each from others. Out of these recordings, 23 were chosen randomly from a set of 4000 recordings, and remaining recordings were selected to represent clinically important arrhythmias. The "MIT-BIH noise stress test database" or "nstdb" [26], [32] contains three recordings of half-hour duration and containing noises typical to ambulatory recordings. It also contains 12 noisy ECG recording of same duration. The noise recordings were taken from physically active volunteers and using standard ECG recorder, leads, and electrodes. Electrodes were placed on limbs of volunteers to acquire the typical noises but without ECG. The three noise records namely baseline wander "bwm", muscle (EMG) artifact "mam", and electrode motion artifact "emm", were collected by selecting intervals that predominantly contain these artifacts. The noisy ECG record were obtained by adding EMG noise to ECG records from "mitdb". The "MIT-BIH atrial fibrillation database" or "afdb" [27], [32] contains 10-hour ECG recordings. The "MIT-BIH ST change database" or "stdb" [28], [32] has 28 ECG recordings of varying length. Out of these, 23 recordings were made during exercise stress test and selected for exhibition of transient ST depression. The last 5 recordings are excerpts of long-term ECG recordings and selected for exhibiting ST elevation.

The "MIT-BIH malignant ventricular ectopy database" or "vfdb" [29], [32] contains 22 half-hour recordings with episodes of sustained ventricular flutter, ventricular fibrillation, and ventricular tachycardia. The "MIT-BIH T-wave alternans challenge database" or "twadb" [30], [32] contains 100 multichannel ECG records from subjects including patients with myocardial infarctions, transient ischemia, ventricular tachyarrhythmias, and other risk factors for sudden cardiac death, as well as healthy controls and synthetic cases with calibrated amounts of T-wave alternans. In most cases, the recordings contain the standard 12 signals, but a few recordings contain only 2 or 3 signals. The "MIT-BIH sudden cardiac death Holter database" or "sddb" [31], [32] contains 23 half-hour excerpts from recordings obtained by Holter monitoring. In these recordings, 4 are from patients who suffered from atrial fibrillation, one is from a patient with continuous pacing of sinus rhythm, and 18 are from patients with underlying sinus rhythm (four with intermittent pacing). Most patients suffered from cardiac arrest and all of them suffered from continuous ventricular tachyarrhythmia. Data regarding drug dosages and regimens and information on patients are unavailable or limited, but these recording are important as they provide unique clues to the pathogenesis regarding sudden death syndrome.

Out of these databases, the arrhythmia database (mitdb) [125] and the noise stress test database (nstdb) [33] are the most commonly used databases in literature on ECG denoising techniques. Both are available with sampling frequency of 360 Hz and 11-bit resolution.

3.3 Subjective evaluation using visual inspection

Subjective evaluation is most commonly carried out by visual inspection of the processed output for smoothness and specific features. Tracey and Miller [33] while evaluating denoising of signals with additive noise by non-local means (NLM) filtering examined the changes in the high-amplitude signal regions and for smoothening of added noise. They observed that denoising did not affect the high-amplitude regions and smoothened the actual signals and signals with added noise. Kania et al [14] examined beat-to-beat changes in ECG shape for evaluating the advantage of a wavelet based technique over averaging of cardiac cycles with use of cross-correlation method. Sayadi and Shamsollahi [34] compared the shape of the output ECG from their denoising method with that of noise-free ECG. When ECG signal was added with real EMG artifact, its shape showed significant changes. After denoising, the ECG signal shape appeared to be restored and diagnostic features in the signal were preserved. Tikkanen [35] reported that the denoising technique removed added noise but introduced distortion in the form of error with a large amplitude (one fifth of original ECG), indicating that the denoising method had seriously altered the ECG signal. Lee and Bein [7] used visual inspection to show that their variable bandwidth filter on ECG signals with additive noise reconstructed the QRS complex, P-wave, and T-wave accurately.

Cherkassy and Kitts [36] used subjective evaluation of a method proposed by them to reduce EMG noise. They showed that after denoising by moving average filter and Chebyshev filter 1, it was not possible to consistently detect P, R, and T waves. After denoising using their technique, ambiguity between P and T waves substantially reduced. Su and Zhao [37] used visual inspection to examine the characteristics of ECG signal like amplitude of R peak, smoothness, impulsive noise, and presence of pseudo-Gibbs phenomena in Q and S waves for comparing the output of their wavelet-based denoising method with those from other methods. Using visual inspection, Li et al [38] showed that severe EMG noise was effectively suppressed by their denoising method.

In summary, the most commonly used subjective evaluations have employed examination of (i) high amplitude of ECG signal, (ii) beat-to-beat relation in shape of ECG signal, (iii) features of ECG having diagnostic importance like QRS complex, P-wave and T-wave, (iv) presence of artifacts like EMG, (v) smoothness of ECG waveform, and (vi) presence of distortions like pseudo-Gibbs phenomenon, etc. The main advantage of the subjective methods is that access to noise-free reference is not essential and it can be used for evaluation of denoising for its clinical usefulness. Being a subjective method, there is scope for large variability in the assessment. Sometimes, distortions in clinically important features may get missed. Despite these shortcomings of quantification and reproducibility, subjective evaluation is essential for assessing the performance of a new denoising technique.

3.4 Objective evaluation based on sample-by-sample comparison of two waveforms

Most of the commonly used objective evaluation methods are based on a sample-by-sample comparison of the denoised signal and the original noise-free signal. Mean square error provides a global measure of the noise in the output signal, treating distortions introduced by the denoising process as additive noise. Several performance indices based on this error estimate have been reported in the literature: root-mean-square error (RMSE), signal-to-noise ratio (SNR), percentage relative distortion (PRD), etc. As dc offsets lower the sensitivity of evaluation of denoising [39], the measures are calculated after removing any dc offsets. Another method involves calculating the correlation coefficient between the time-aligned samples of two waveforms, removing any dc offsets in the definition of the coefficient. These measures can be easily calculated and provide a reproducible quantification of the performance. They do not distinguish between residual noise and the error introduced due to distortion during denoising. Unlike visual inspection, these measures do not indicate the specific sections of ECG which are contaminated with noise or the signal features which have suffered distortion, and give an overall indication of the presence or absence of noise or error. They give equal weight to the error in the entire waveform. It is to be noted that removal of any noise present in the reference signal gets reported as error. Hence the reference signals used for objective method should be free of noise.

3.4.1 Mean-squared error (MSE) and root-mean-squared error (RMSE)

Mean-squared error (MSE) in a signal $x(i)$ is the average power of error in it with respect to noise-free reference signal $s(i)$ and is given as the following:

$$\mathrm{MSE} = \frac{1}{N}\sum_{i=1}^{N}\left(x(i) - s(i)\right)^2 \tag{3.1}$$

The root-mean square error (RMSE) is the square root of MSE. Both the measures have been used in many studies on ECG denoising [14], [3], [40], [22], [41]. For evaluation using this method, MSE in the noisy input signal and that in the denoised signal are calculated separately, and used for reporting improvement in MSE. Alternatively, MSE in the outputs of different denoising methods are reported for comparing them.

Cherkassy and Kitts [36] used MSE for evaluation of their wavelet-based denoising method on ECG records of approximately 16 s duration, with about one-fourth of it marked as clean signal and about one-fourth as noisy signal. For calculating MSE for the denoised signal corresponding to the noisy section, the reference signal was obtained by a careful visual alignment of the clean signal section. Tracey and Miller [33] used MSE for evaluating the denoising method based on non-local means and reference denoising technique, like thresholding or ICA based wavelets, on noisy signals with different SNRs obtained by adding Gaussian noise to seven ECG signals from "mitdb".

3.4.2 Signal-to-noise ratio (SNR)

SNR is widely used in evaluation of denoising techniques. With noise-free reference signal $s(i)$ and noisy signal $x(i)$, SNR is given as the following:

$$\mathrm{SNR} = 10\log\left(\frac{\sum_{i=1}^{N}\left(s(i)\right)^2}{\sum_{i=1}^{N}\left(x(i)-s(i)\right)^2}\right) \tag{3.2}$$

For a denoising system with noise-free reference signal $s(i)$, input signal $x(i)$, and processed output signal $y(i)$, the input and output SNRs are given as the following:

$$\mathrm{SNR_{in}} = 10\log\left(\frac{\sum_{i=1}^{N}\left(s(i)\right)^2}{\sum_{i=1}^{N}\left(x(i)-s(i)\right)^2}\right) \tag{3.3}$$

$$\mathrm{SNR}_{\mathrm{out}} = 10\log\left(\frac{\sum_{i=1}^{N}(s(i))^2}{\sum_{i=1}^{N}(y(i)-s(i))^2}\right) \tag{3.4}$$

The improvement in SNR is given as

$$\mathrm{SNR}_{\mathrm{impr}} = 10\log\left(\frac{\sum_{i=1}^{N}(x(i)-s(i))^2}{\sum_{i=1}^{N}(y(i)-s(i))^2}\right) \tag{3.5}$$

This method has been used by many investigators [3], [16]-[18], [20], [21], [33], [34] [37]- [39], [41]-[46]. $\mathrm{SNR}_{\mathrm{out}}$ or $\mathrm{SNR}_{\mathrm{impr}}$ is generally plotted as a function of $\mathrm{SNR}_{\mathrm{in}}$, for a particular combination of signal and additive noise to assess the extent and range of effectiveness of the denoising methods.

Rahman et al [3] used $\mathrm{SNR}_{\mathrm{impr}}$ to compare their denoising technique using normalized signed regression LMS with existing LMS technique. The noisy test signals were obtained by adding ECG recordings from "mitdb" and noises from "nstdb". Sayadi and Shamsollahi [34] compared extended Kalman filter with 17 parameters (EKF17) and that with 2 parameters (EKF2) by finding $\mathrm{SNR}_{\mathrm{impr}}$ for noisy test signals obtained by addition of signal and noise records (from the databases nsrdb, mitdb, nstdb) ECG corrupted with EMG at 2 dB SNR, ECG corrupted white Gaussian noise at of 5 and -2 dB SNR, and ECG corrupted with motion artifact at 5 dB SNR. Sameni et al [46] used noisy test signals obtained from records of "nsrdb" and "nstdb" to evaluate the performance of their denoising technique for removing muscle artifacts. Velasco et al [20] compared their proposed EMD technique with Butterworth filtering and wavelet denoising techniques with noisy test signals obtained using 5 ECG records from "mitdb" and the noise record from "nstdb" and white Gaussian noise. Tracey and Miller [33] used 7 ECG signals from "mitdb", and white Gaussian noise to get noisy signals with different desired SNRs to test denoising technique proposed by them based on non-local means.

3.4.3 Percentage RMS difference (PRD)

PRD is one of the widely used methods in evaluation of compression algorithms [39], [45]. It is the ratio of root mean square value of the error in the noisy signal with reference to the reference signal to the root mean square value of the reference signal. With the signal $x(i)$ and noise-free reference $s(i)$, PRD is given as the following:

$$\mathrm{PRD} = 100\frac{\left(\frac{1}{N}\sum_{i=1}^{N}\left(x(i)-s(i)\right)^2\right)^{0.5}}{\left(\frac{1}{N}\left(\sum_{i=1}^{N}\left(s(i)\right)^2\right)\right)^{0.5}} \tag{3.6}$$

We see that PRD is normalized RMSE. Treating the error as noise, PRD and SNR are directly related as the following:

$$\mathrm{SNR} = -20\log(\mathrm{PRD}/100) \tag{3.7}$$

For evaluation of a denoising technique, PRDs of the input and denoised signals are calculated and reduction in PRD due to denoising is reported. For comparing several methods, PRDs of outputs of these methods may be compared. Tracey and Miller [33] used this method for evaluation of denoising, proposed by them based on non-local means, using noisy test signals with different SNRs generated with ECG signals from "mitdb" (records: 100, 103-106, 115, 215) and white Gaussian noise.

3.4.4 Correlation coefficient

The correlation coefficient or the normalized correlation is the projection of one signal on the other and it is a measure of the similarity between the two signals. The correlation coefficient for test signal $x(n)$ and reference signal $s(n)$ is given as the following:

$$r_{xs} = \frac{[\sum_{n=1}^{N}(x(n)-\bar{x})(s(n)-\bar{s})]/N}{\sqrt{[\sum_{n=1}^{N}(x(n)-\bar{x})^2]/N}\sqrt{[\sum_{n=1}^{N}(s(n)-\bar{s})^2]/N}} \tag{3.8}$$

where $\bar{x}$ and $\bar{s}$ are the mean values i.e. dc values of $x(n)$ and $s(n)$ respectively. For evaluation of a denoising technique, correlation coefficient between the output and reference can be compared with that between input and reference.

In case the signal and noise waveforms are uncorrelated, there is a one-to-one relationship between SNR and correlation coefficient. Let us assume that the noisy signal $x(n)$ is a sum of noise-free signal $s(n)$ and noise $d(n)$ as the following

$$x(n) = s(n) + \alpha d(n) \tag{3.9}$$

Let us further assume that $s(n)$ and $d(n)$ have zero dc value and have the RMS value of σ, with the SNR value of $x(n)$ given as

$$\mathrm{SNR} = -20\log\alpha \tag{3.10}$$

Assuming $s(n)$ and $d(n)$ to be uncorrelated, the RMS value of $x(n)$ is given as

$$\text{RMS}(x) = (1+\alpha^2)^{1/2}\sigma \tag{3.11}$$

Correlation coefficient in this case can be given as

$$r_{xs} = \frac{[\sum_{n=1}^{N}(s(n)+\alpha d(n))s(n)]/N}{(1+\alpha^2)^{1/2}\sigma^2}$$

which can be simplified as following:

$$r_{xs} = 1/(1+\alpha^2)^{1/2} \tag{3.12}$$

Thus Equations 3.10 and 3.12 shows us that SNR and r_{xs} are related by following by the following equation:

$$SNR = 10\log\left(r_{xs}^{\ 2}/\left(1-r_{xs}^{\ 2}\right)\right) \tag{3.13}$$

Wu et al [14] used correlation coefficient for evaluation of denoising of ECG using adaptive filters, with noisy test signals obtained by using seven ECG records from "mitdb" and real muscle artifact from "nstdb". Similar noisy signals were used by Foresta et al [10] to evaluate ECG denoising by wavelet-ICA filter.

It may be noted that the correlation coefficient, like RMSE and SNR, gives similar emphasis to all segments of the waveform. For example, a small change in R-peak will lower the *r* value considerably, but it may not be significant diagnostically. A similar reduction in *r* value due to distortion in regions of ST or P-peak may affect diagnostic value of the signal.

3.5 Objective evaluation based on waveform features

Some of the evaluation methods are based on statistical measures or features of diagnostic significance. These methods have not been widely used in ECG denoising literature, but may give a better indication of clinical usefulness of the denoising method.

3.5.1 Improvement index based on signal statistics

Tong et al [8] used improvement index (II) based on L2-norm and max-min for evaluation of their adaptive filtering technique for suppression of motion artifact. L2-norm of a signal $x(n)$ is quantification of its energy and is given as the following:

$$\text{L2}(x) = \sqrt{\sum_{i=1}^{N}(x(i)^2)} \tag{3.14}$$

while max-min is its peak-to-peak amplitude and is given as the following:

$$\mathrm{MM}\{x\} = \max_{\forall n}[x(n)] - \min_{\forall n}[x(n)] \quad (3.15)$$

Improvement index is calculated from L2-norm or MM values as the following:

$$\mathrm{II} = \frac{\text{Pre-denoising value} - \text{Post-denoising value}}{\text{Pre-denoising value} - \text{Artifact-free value}}$$

For noise-free reference $s(i)$, noisy input $x(i)$, and processed output signal $y(i)$, the improvement indices can be give as the following:

$$\mathrm{II(L2)} = \frac{\mathrm{L2}(x) - \mathrm{L2}(y)}{\mathrm{L2}(x) - \mathrm{L2(s)}} \quad (3.16)$$

$$\mathrm{II(MM)} = \frac{\mathrm{MM}(x) - \mathrm{MM}(y)}{\mathrm{MM}(x) - \mathrm{MM(s)}} \quad (3.17)$$

A value of II significantly less than one indicates ineffective denoising, and that near to one indicates effective denoising. A value significantly greater than one indicates distortions introduced by the denoising technique. Negative value of II indicates severe distortion introduced by denoising. If II is near to one for L2 and MM both, denoising technique can be considered to be highly effective.

Improvement index can also be calculated for several other statistical measures on the signal to get a higher degree of confidence on the denoising method. For example, we can apply it on the third and forth moments of the signals i.e. skewness and kurtosis respectively. Improvement indices near to one for all of them may indicate effective denoising. It may be further noted that if the selected statistical measure is reasonably constant over different segments of the noise-free signal, II can be calculated without having access to the reference signal corresponding to the segment being denoised. Thus it can serve as a quantitative measure of denoising in cases where we do not have noise-free reference signal available.

3.5.2 Errors in R-peak detection

Objective evaluation based on errors in R-peak detection can be useful for certain applications like arrhythmia detection. For this purpose, an R-peak detection technique is applied on the test signal and the detected cardiac cycles are compared with those detected by application of the technique on the noise-free reference. A reduction in the errors after denoising is used as an indicator of its effectiveness. The R-peak detection technique used for this purpose should be selected so that it is neither insensitive nor oversensitive to the presence of artifacts. Pan-Tompkins technique [47] has been used by Mithun et al [4] for evaluation of their wavelet-based denoising technique with noisy test signals generated by adding ECG-free motion artifacts and EMG noise to noise-free ECG and at different SNRs.

3.6 Some proposed objective methods

In this section, some objective methods which can serve as supplementary to those described earlier are proposed. In addition to the improvement indices based on L2-norm and max-min of the signal waveform, we can use improvement indices based on higher statistical moments of the signal, like skewness and kurtosis. Error in cumulative distribution function (CDF) can be used as another measure. In the R-peak detection based evaluation, the comparison between the cardiac cycles detected from the test signal and those from the reference signal is carried out visually. This method can be made more objective and more informative by an automated comparison of the detected R-peak in the test signal with those in the reference signal as a function of temporal tolerance.

3.6.1 Improvement index based on skewness and kurtosis

Corruption of a signal by noise may alter its statistical properties. After denoising, the statistical properties of the processed output can be expected to be close to that of the noise-free signal. Earlier in Section 3.5.1, II based on L2-norm and max-min has been described. It is proposed to also calculate it for two higher statistical moments: skewness and kurtosis.

Skewness [48], [49] is a measure of asymmetry of amplitude distribution of signal and it can be calculated from the signal samples $x(n)$ as the following:

$$\text{Skewness } = \left[\frac{1}{N} \sum_{n=1}^{N} \left(x(n) - \bar{x} \right)^3 \right] / \sigma^3 \tag{3.18}$$

where $\bar{x}$ is mean and σ is the ac RMS, and the two are given as the following:

$$\bar{x} = \frac{1}{N} \sum_{n=1}^{N} x(n)$$

$$\sigma = \sqrt{\frac{1}{N} \sum_{n=1}^{N} \left(x(n) - \bar{x} \right)^2}$$

Skewness for signals with Gaussian distribution, uniform distribution or other types of symmetric amplitude distribution is zero. Negative value for the skewness indicates that the amplitude distribution of the signal is skewed to the left and a positive value indicates that it is skewed to right.

Kurtosis [48]-[50] is a measure of the peakiness of the amplitude distribution of the signal and is calculated from the signal samples as the following:

$$\text{Kurtosis} = \left[\frac{1}{N} \sum_{n=1}^{N} \left(x(n) - \bar{x} \right)^4 \right] / \sigma^4 \tag{3.19}$$

A Gaussian distribution has kurtosis of 3 [50] and a uniform distribution has kurtosis of 1.8. The distribution with high kurtosis is characterized by heavy tails and sharp peak near the mean, while that with low kurtosis is characterized by flat top near the mean. However, II values based on these may be taken as indication of denoising.

II can be calculated based on skewness and kurtosis using Equation 3.15, as the following:

$$\mathrm{II(skew)} = \frac{\mathrm{skew(x)} - \mathrm{skew}(y)}{\mathrm{skew}(x) - \mathrm{skew(s)}} \tag{3.20}$$

$$\mathrm{II(kurt)} = \frac{\mathrm{kurt(x)} - \mathrm{kurt}(y)}{\mathrm{kurt}(x) - \mathrm{kurt(s)}} \tag{3.21}$$

II values for L2-norm, max-min, skewness, and kurtosis being close to one can be taken as an indication of effective denoising. It may be noted that the values of skewness and kurtosis have simple physical interpolation for signal with mono modal distribution (distribution with single peak). However, II based on skewness and kurtosis may be useful in all cases.

3.6.2 Error in cumulative distribution function (E-CDF)

Cumulative distribution function (CDF) is one of the characteristics of a signal. and addition of noise is expected to alter it. After denoising, CDF can be expected to be close to that of the noise-free signal. For quantification, RMS of error in CDF can be calculated as following:

$$\text{E-CDF}_{(x)} = \left[\sum_{k=1}^{K} \left(\mathrm{CDF}_x(a_k) - \mathrm{CDF}_s(a_k) \right)^2 / K \right]^{1/2} \tag{3.22}$$

where $\mathrm{CDF}_s(a_k)$ and $\mathrm{CDF}_x(a_k)$ are CDFs for noise-free signal s and noisy signal x, respectively, for amplitude bin $a_{k,}$ and K is the total number of bins. As the CDF values have a range of 0 – 1, the E-CDF values also have a range of 0 – 1. E-CDF can be calculated for the output as

$$\text{E-CDF}_{(y)} = \left[\sum_{k=1}^{K} \left(\mathrm{CDF}_y(a_k) - \mathrm{CDF}_s(a_k) \right)^2 / K \right]^{1/2} \tag{3.23}$$

3.6.3 Decomposition of denoising error

Errors in the output of a denoising technique may be contributed by three components: attenuation of input signal, residual noise due to imperfect suppression of input noise, and distortions introduced by the denoising technique. Decomposition of the error into these components may be useful in comparing the performance of different denoising techniques and in improving the techniques. It is proposed that correlation of the output waveform with the input signal and input noise can be used for estimating the signal attenuation, residual noise, and distortion separately.

Let us assume that the noisy signal $x(n)$ is a sum of signal $s(n)$ and noise $d(n)$, as given earlier in Equation 3.9, as the following:

$$x(n) = s(n) + \alpha d(n)$$

with $s(n)$ and $d(n)$ having zero dc value and the RMS value of σ. The output $y(n)$ after application of the denoising technique can be modeled as the sum of scaled input signal, attenuated input noise, and a distortion component, as the following:

$$y(n) = \beta s(n) + \gamma \alpha d(n) + \kappa e(n) \tag{3.24}$$

where β is the scaling coefficient for the input signal $s(n)$, γ is the attenuation coefficient for the input noise $\alpha d(n)$, and $\kappa e(n)$ is the distortion coefficient with $e(n)$ having the RMS value of σ. Values of these coefficients the unprocessed signal are as β = 1, γ = 1, and κ = 0. Perfect denoising, corresponds to β = 1, γ = 0, and κ = 0.

The RMS value of the noisy input is given as

$$\sigma_x = \left(1 + \alpha^2\right)^{1/2} \sigma \tag{3.25}$$

Correlation of the noisy input $x(n)$ with the reference signal $s(n)$, is given in accordance with Equation 3.12 as

$$r_{xs} = 1/\left(1 + \alpha^2\right)^{1/2} \tag{3.26}$$

and that with the noise $d(n)$ is given as

$$r_{xd} = \alpha/\left(1 + \alpha^2\right)^{1/2} \tag{3.27}$$

The output RMS is given as

$$\sigma_y = \left(\beta^2 + \gamma^2 \alpha^2 + \kappa^2\right)^{1/2} \sigma \tag{3.28}$$

Correlation of the denoised output $y(n)$ with reference signal $s(n)$ is given as

$$r_{ys} = \beta/\left(\beta^2 + \gamma^2 \alpha^2 + \kappa^2\right)^{1/2} \tag{3.29}$$

and that with the noise $d(n)$ is given as

$$r_{yd} = \gamma\alpha/\left(\beta^2 + \gamma^2 \alpha^2 + \kappa^2\right)^{1/2} \tag{3.30}$$

From Equations 3.28, 3.29, and 3.30, we get the three coefficient values as the following:

$$\beta = r_{ys}\sigma_y/\sigma \tag{3.31}$$

$$\gamma = r_{yd}\sigma_y/(\alpha\sigma) \tag{3.32}$$

$$\kappa = \left(\sigma_y^2/\sigma^2 - \beta^2 - \gamma^2\alpha^2\right)^{1/2} \tag{3.33}$$

3.6.4 R-peak detection as a function of temporal tolerance

For using improvement in R-peak detection for evaluation of the denoising methods, the R-peak detection technique should neither be insensitive nor oversensitive to the noise. Pan-Tompkins technique [47] has been earlier found to be suitable for this purpose [4], [17], [18]. The R-peak detection rate is calculated by visually examining the number of beats correctly detected in the denoised signal with respect to the beats identified in the noise-free reference. It is proposed that we can apply an automated method by comparing the locations of detected R-peaks in the test signal with those in the reference signal and errors may be measured as a function of temporal tolerance, i.e. the tolerance in the alignment of the peaks in the test signal with those in the reference signal. For a temporal tolerance τ_a, a detection error refers to missing a true peak, i.e. when no peak is detected in the test signal within $\pm\tau_a$ interval around an R-peak in the noise-free signal. Similarly, an insertion error refers to detecting a false peak, i.e. when there is no R-peak in the noise-free signal within $\pm\tau_a$ interval around an R-peak detected in the test signal. Detection and insertion errors can be given as a fraction of the number of R-peaks in the noise-free signal as the following:

$$\text{Detection Error}(\tau_a) = \frac{\text{No. of missed R-peaks}}{\text{No. of R-peaks in noise-free ECG}} \tag{3.34}$$

$$\text{Insertion Error}(\tau_a) = \frac{\text{No. of inserted R-peaks}}{\text{No. of R-peaks in noise-free ECG}} \tag{3.35}$$

Both errors are calculated as a function of temporal tolerance and are expected to serve as a useful indicator of the effectiveness of denoising methods. It may be noted that the detection error has an upper bound of 1 which corresponds to all true peaks being missed by the detection method. There is no theoretical upper bound on insertion errors. However, both errors being less than 1% for temporal tolerance of 10 ms may be considered as effective denoising for arrhythmia detection.

3.7 Investigations on the objective evaluation methods based on statistical measures

This section presents investigations on the objective evaluation methods based on statistical measures as described earlier. These were carried out with noisy test signals generated by adding noise-free signal $s(n)$ and scaled version of noise $d(n)$ with different values of SNR. Synthesized waveforms and actual ECG recordings were used as noise-free signals. Synthesized white Gaussian random waveform and ECG-free artifact recording were used as noise.

The first set of investigations were conducted to examine the effect of noise on the statistical measures using, 1 Hz sine wave, 10 Hz sine wave, 1 Hz square wave, and white Gaussian random waveform as the test signals and another white Gaussian random waveform as the noise. The second set of investigations were carried out on ECG signals. Noise-free ECG signals were taken from the records 105,106, and 107 of the database "mitdb" [25] . Further, three periodic ECG test waveforms were generated by repetitive concatenation of typical ECG cycles (one waveform each corresponding to a cycle from each of the records). Twenty noise-free ECG records were used as actual ECG waveforms. Three ECG-free artifacts were taken from the database "nstdb" [26]: motion artifact (emm), EMG noise (mam), and baseline wander (bwm). In addition to these, white Gaussian random waveform was also used as noise. The four types of interferences (three artifacts and white Gaussian random waveform) were also used as signals to get their statistical measures. All waveforms in both sets of investigations had sampling frequency of 360 Hz and duration of 1 minute. They were normalized to have RMS values of 1 to generate the noisy signals of 1 minute duration using Equations 3.9 and 3.10 with SNR of ∞ (no noise), +12, +9, +6, +3, 0, −3, −6, −9, and −12 dB.

All test waveforms were normalized to have RMS value of 1. Statistical parameters i.e. max-min, skewness and kurtosis, of the waveforms are given in Table 3.2. For ECG waveforms, the max-min values are similar to that for white Gaussian noise or much larger, skewness is distributed from low negative values to positive values of up to 4, values of kurtosis range from values similar to that for white Gaussian noise to almost exceeding 20. Statistical parameters for different noise-free segments of an ECG record are generally not very different, indicating that different ECG records can be characterized by a set of these parameters. Figure 3.1(A) displays CDF for square wave, sine wave and white Gaussian noise. Figure 3.1(B) shows CDF for different ECG records, with the curve shapes having some variation but a general similarity to that of white Gaussian noise. The curves for artifacts are shown in Figure 3.1(C) and it is seen that they generally have much slower transitions than the curves for ECG records. The signal and noise waveform as described above were used to generate test waveforms. As all the waveforms were normalized to RMS values of 1, noisy test wave were generated using Equation 3.9 and 3.10 to generate noisy test waveforms

As all the test waveforms are normalized to have RMS of one and noisy waveforms are obtained by multiplying noise waveform with scaling factor α and adding it to signal waveform. Therefore, we get SNR = $-20\log\alpha$, RMSE = α, RMS = $\sqrt{1+\alpha^2}$, and correlation coefficient r = $1/\sqrt{1+\alpha^2}$. The results are given in Tables 3.3(A) – 3.3(D) for 1 Hz sine wave, 10 Hz sine wave, 1 Hz square wave, and white Gaussian noise, respectively, as the noise-free test signals, with white Gaussian noise as the additive noise at different SNRs. Values of RMSE, RMS, max-min, skewness, kurtosis, r, E-CDF are given as function of SNR. The means and standard deviations of these parameters were calculated for noisy waveforms generated using 10 different white Gaussian noises. The values in the tables approximately conform to the calculated values of relations. The values of r show a small deviation from the calculated ones at lower SNRs. The values of the max-min are waveform dependent, but they become waveform independent at very low SNRs. Max-min does not exhibit a very large standard deviation, which can be attributed to stationary nature of noise. As the signal and noise waveforms have a skewness of zero, skewness does not change with SNR. Kurtosis of noisy signal approaches to that of noise as SNR decreases. E-CDF increases with decrease in SNR for sine and square waves. This is expected because sine, square, and white Gaussian waveforms have very different CDF functions as seen in Figure 3.1(A). When signal and noise are white Gaussian waveforms, E-CDF remains zero independent of SNR. Thus use of all these parameters together may be helpful in assessing the performance of noise suppression techniques.

The results for noisy signal obtained by addition of ECG waveforms as the signal and white Gaussian waveform as the noise are given in Table 3.4. It is noted that although the values of max-min, skewness, and kurtosis of different ECG signals are different, they all tend to reach similar values at lower SNRs. Periodic ECG signals were obtained by repeated concatenation of single ECG cycles. The results for such periodic signals are given in Table 3.5. The parameters for these waveforms show the same patterns as for actual ECG signals, indicating that the statistical parameters are not significantly affected by heart rate variability. It has been observed earlier based on the values in Table 3.2 that statistical parameters for different noise-free segments of an ECG record are generally not very different, although those for segments from different records may be different. These two observations together indicate that improvement indices for these parameters can be calculated without having access to the reference signal corresponding to the segment being denoised. Thus the improvement indices can serve as a quantitative measure of denoising in cases where the noise-free reference signals are not available.

In order to study the differences between the statistical properties of artifacts and white noise, the statistical parameters were calculated for noisy waveforms generated using the artifacts as the signal and white Gaussian random waveform as the noise. In Table 3.6 initial one-minute segment signal-free artifacts from database "nstdb" were used as signals, with white Gaussian noise as the additive noise at different SNRs. The results for subsequent segments are qualitatively similar. The max-min, skewness, and kurtosis values for BW and motion artifact are similar to the corresponding values for noise, while they are generally higher in case of EMG. With addition of noise, the statistical parameters change towards the corresponding values for the noise.

For the results in Table 3.7, three ECG records from database "mitdb" were used as signal and EMG artifact 'mam' form database "nstdb" was used as noise at different SNRs. Max-min and kurtosis were maximum for ECG-106 and minimum for ECG-107. For the results in Table 3.8, three ECG records from database "mitdb" were used as signal and motion artifact 'emm' from database "nstdb". as noise at different SNRs The values of E-CDF as well as *r* for ECG-105 are more than the corresponding values for ECG-107, clearly indicating that examination of both parameters is needed for assessment of denoising. For the results in Table 3.9, three ECG records from database "mitdb" were used as signal and BW artifact 'bwm' form database "nstdb" was used as noise at different SNR. These results also show that low values of E-CDF may be useful as an additional indicator of denoising.

To get a verification of the calculation of the performance indices β, γ, and κ, these were calculated for the noisy signals. The mean and standard deviation of the indices for different noise and SNR combinations are given in Table 3.10. As noise addition does not introduce any signal attenuation or distortion, the theoretical values of β, γ, κ, are 1, 1, and 0, respectively. The means of the calculated values for ECG with white Gaussian noise are very close to the corresponding theoretical values and the standard deviations are very small. For ECG with BW, EMG, and MA, the means of the calculated values show some deviations from the corresponding theoretical values and the standard deviations are somewhat larger than the corresponding values for white noise. It may be noted that the calculated values of κ were imaginary in a few cases and magnitudes of the values were used in calculation of mean and standard deviation. These results show that while white noise was uncorrelated with the ECG signals, the other artifacts only approximately met the criterion for uncorrelated noise.

3.8 Investigations on the temporal tolerance of R-peak detection

This section presents investigations on temporal tolerance of R-peak detection in noisy ECG signals as a function of SNR and type of noise. Noise-free ECG segments of one-minute duration each were selected from the records 105, 106, and 112 of the database "mitdb" [25] . Three ECG-free artifacts were taken from the database "nstdb" [26]: motion artifact (emm), EMG noise (mam), and BW artifact (bwm). In addition to these, white Gaussian random waveform was also used as noise. All waveforms had sampling frequency of 360 Hz. They were normalized to have RMS values of 1 to generate the noisy signals of 1 minute duration using Equations 3.9 and 3.10, with SNR values of ∞ (no noise), +12, +6, 0, −6, and −12 dB. Pan-Tompkins algorithm [47] was used for R-peak detection. The results are in the form of detection and insertion errors with reference to the position of the detected R-peaks in the noise-free signal and as defined in Equation 3.34 and Equation 3.35 as a function of temporal tolerance (0 to 100 ms in 10 ms steps) for different values of SNR.

The results are given in Tables 3.11 − 3.14. Generally both types of errors increase with decrease in SNR. For a given signal and noise at a specific SNR, the errors generally decrease with increase in the temporal tolerance. At higher SNRs (12 and 6 dB) and at low tolerance, detection and insertion errors are very often equal. At these SNRs, there are almost no false detections and almost all R-peaks get detected although with some misalignments. As the detection and insertion errors are defined with respect to a specific tolerance, each misalignment beyond the tolerance limit contributes to a detection as well as an insertion error, and thus the two errors become equal. It is also seen that the Pan-Tompkins algorithm for R-peak detection has a very low sensitivity to BW and the errors for this artifact are much lower than those for other artifacts.

In introducing insertion errors, white Gaussian noise is ranked as the highest, followed by EMG, motion artifact, and BW. In introducing detection errors, EMG is ranked as the highest, followed by motion artifact and white Gaussian having nearly similar effects, and BW as the lowest.

3.9 Summary

In this chapter, after a description of some of the ECG databases commonly used in the evaluation of ECG denoising techniques, several subjective and objective methods of evaluation have been reviewed. Subsequently a few supplementary methods for objective evaluation have been proposed. The parameters and indices used in these methods have been investigated by computing them for different combinations of signals and artifacts as a function of SNR.

In our investigations on evaluation of denoising techniques, the denoising techniques will be applied on noisy test signals generated by adding noise-free ECG signals and artifacts from the MIT-BIH database. It will permit us to use both subjective and objective methods. Access to noise-free reference will help in relating the results of these methods. Subsequently the techniques will be applied for processing Holter recordings without access to noise-free references. In these cases, we will be using subjective methods. In addition, we will use improvement indices based on statistical measures obtained from visually selected noise-free segments.

As stated earlier in Section 3.3, subjective methods use visual inspection and suffer from a scope for large variability in the assessment results but they are essential for assessing the performance of denoising techniques particularly in the absence of noise-free reference signals. Based on the earlier studies in the literature, it is proposed to use subjective evaluation by visual inspection for (i) change in ECG peaks, (ii) restoration of beat-to-beat relation in shape of ECG signal, (iii) enhancement of features like QRS complex, P-wave and T-wave, (iv) presence of artifacts like EMG, (v) smoothness of ECG waveform, and (vi) presence of distortions like pseudo-Gibbs phenomenon introduced in the denoising process.

We will be using a mix of established objective methods along with the proposed ones. From among the commonly used objective methods in the literature, we will be using SNR improvement and correlation coefficient. As proposed in Section 3.6.3, we will be using correlation of the output signal with the noise-free reference and the artifact to examine the components of the error in the denoised output by calculating signal attenuation (β), residual noise (γ), and distortion component (κ). This decomposition is expected to be helpful in understanding the effect of processing parameters of the denoising technique on different components of the error and in selecting an optimal set of parameters. Further, in case of multiple types of additive artifacts, correlation of the output with individual artifacts can be used in examining the effect of the parameters in terms of residual noises and distortions. We also will be using improvement indices based on max-min, L2-norm, skewness, and kurtosis and RMS error in cumulative distribution function (E-CDF). To assess the usefulness of the denoising techniques in arrhythmia detection, we will use errors in R-peak detection using Pan-Tomkins algorithm as a function of temporal tolerance,

Table 3.2: Statistical parameters of test waveforms, all with RMS = 1.0

Test waveform	Max-min	Skewness	Kurtosis
1 Hz sine	2.828	0.000	1.499
10 Hz sine	2.828	0.000	1.500
1 Hz square	2.000	-0.001	1.000
White Gaussian random	6.951	0.009	2.915
ECG-105_00	8.892	2.914	13.063
ECG-105_04	8.608	2.864	12.684
ECG-105_10	8.291	2.861	12.371
ECG-106_ 00	10.124	3.504	20.712
ECG-106_04	9.190	3.230	16.892
ECG-106_10	9.584	3.808	23.720
ECG-106_20	8.425	2.457	12.103
ECG-107_00	7.224	-0.297	3.982
ECG-107_04	6.629	-0.139	3.935
ECG-107_10	6.325	-0.070	3.493
ECG-109_00	7.566	1.384	6.439
ECG-109_04	6.472	1.779	7.399
ECG-109_10	6.112	1.852	7.682
ECG-109_15	6.242	1.886	7.907
ECG-109_20	6.171	1.863	7.777
ECG-109_25	7.535	1.587	6.946
ECG-111_00	9.403	1.394	7.010
ECG-112_00	7.738	1.231	6.631
ECG-105A concat. cycle	5.531	2.833	11.763
ECG-105B concat. cycle	5.501	2.834	11.706
ECG-105C concat. cycle	8.892	2.914	13.063
ECG106A concat. cycle	7.485	2.080	8.735
ECG-106B concat. cycle	6.160	1.669	6.188
ECG-106C concat. cycle	6.429	3.451	17.130
ECG-107A concat. cycle	7.595	-0.004	3.076
ECG-107B concat. cycle	4.360	0.434	2.268
ECG-107C concat. cycle	4.745	-0.727	3.383
ECG 203_00	9.271	1.337	6.232
ECG 209_00	10.414	2.132	13.984
ECG 209_04	10.521	1.706	10.539
ECG 209_10	9.450	2.626	13.674
ECG 209_20	10.315	1.842	11.497
ECG 215_00	12.666	1.493	11.006
Baseline wander bwm_00	5.209	-0.520	3.030
Baseline wander bwm _10	5.436	-0.800	3.590
Baseline wander bwm_15	5.586	0.098	4.552
Baseline wander bwm _25	5.103	-0.785	5.251
Muscle artifact mam _00	10.320	0.601	6.112
Muscle artifact mam _10	7.267	-0.051	2.656
Muscle artifact mam _15	8.534	0.470	5.091
Muscle artifact mam_ 25	7.015	1.037	6.535
Electrode motion emm _00	4.693	0.154	2.902
Electrode motion emm _ 10	5.981	0.987	3.871
Electrode motion emm _ 15	6.625	1.238	5.810

Electrode motion emm _ 25	6.828	0.503	3.472

[ECG-nnn_mm: One-minute ECG signal taken from ECG record "nnn" of "mitdb" starting at "mm" minute in record. Conc. cycle: ECG waveform formed by 90 repetitions of one ECG cycle of 240 samples.]

Table 3.3: Statistical measures of the noisy test signals with synthesized waveforms. Mean & standard deviation (SD) for 10 noise records.

A) signal: 1 Hz sine, noise: white Gaussian random

SNR	RMSE	RMS	Max-min		Skewness		Kurtosis		Corr.Coeff.		E-CDF	
(dB)	Mean	Mean	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
∞	0.000	1.000	2.828	0.000	0.000	0.000	1.499	0.000	1.000	0.000	0.000	0.000
12	0.251	1.032	4.420	0.136	0.002	0.008	1.671	0.006	0.970	0.000	0.088	0.028
9	0.355	1.063	5.122	0.197	0.003	0.010	1.813	0.011	0.943	0.000	0.118	0.030
6	0.501	1.121	6.122	0.281	0.004	0.013	2.035	0.019	0.895	0.001	0.147	0.028
3	0.708	1.229	7.555	0.388	0.004	0.016	2.325	0.033	0.817	0.002	0.175	0.025
0	1.000	1.418	9.630	0.507	0.004	0.019	2.614	0.049	0.709	0.004	0.199	0.024
−3	1.413	1.735	12.569	0.673	0.001	0.021	2.824	0.064	0.581	0.006	0.220	0.023
−6	1.995	2.237	16.742	0.904	-0.002	0.021	2.932	0.073	0.452	0.008	0.248	0.023
−9	2.818	2.996	22.701	1.199	-0.005	0.021	2.975	0.077	0.339	0.009	0.279	0.023
−12	3.981	4.110	31.211	1.588	-0.007	0.020	2.989	0.078	0.249	0.010	0.307	0.024

B) signal: 10 Hz sine, noise: white Gaussian random

SNR	RMSE	RMS	Max-min		Skewness		Kurtosis		Corr.Coeff.		E-CDF	
(dB)	Mean	Mean	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
∞	0.000	1.000	2.828	0.000	0.000	0.000	1.500	0.000	1.000	0.000	0.000	0.000
12	0.251	1.031	4.399	0.066	-0.003	0.007	1.672	0.007	0.970	0.000	0.101	0.030
9	0.355	1.062	5.079	0.093	-0.005	0.009	1.815	0.009	0.942	0.000	0.131	0.031
6	0.501	1.119	6.041	0.137	-0.006	0.012	2.039	0.012	0.894	0.001	0.161	0.030
3	0.708	1.226	7.423	0.181	-0.008	0.015	2.330	0.017	0.816	0.002	0.190	0.027
0	1.000	1.415	9.401	0.243	-0.010	0.017	2.619	0.024	0.708	0.003	0.211	0.025
−3	1.413	1.732	12.232	0.384	-0.010	0.019	2.825	0.032	0.579	0.005	0.232	0.023
−6	1.995	2.233	16.297	0.546	-0.010	0.019	2.931	0.037	0.449	0.006	0.260	0.021
−9	2.818	2.992	22.189	0.737	-0.010	0.020	2.972	0.038	0.336	0.007	0.290	0.018
−12	3.981	4.106	30.611	1.075	-0.010	0.021	2.986	0.037	0.245	0.008	0.317	0.016

C) signal: 1 Hz square, noise: white Gaussian random

SNR	RMSE	RMS	Max-min		Skewness		Kurtosis		Corr.Coeff.		E-CDF	
(dB)	Mean	Mean	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
∞	0.000	1.000	2.000	0.000	-0.001	0.000	1.000	0.000	1.000	0.000	0.000	0.000
12	0.251	1.031	3.762	0.089	-0.001	0.003	1.231	0.002	0.970	0.000	0.483	0.030
9	0.355	1.061	4.490	0.125	0.000	0.005	1.423	0.005	0.942	0.000	0.544	0.018
6	0.501	1.119	5.517	0.177	0.001	0.008	1.724	0.011	0.894	0.001	0.594	0.010
3	0.708	1.226	6.967	0.250	0.003	0.013	2.116	0.022	0.816	0.002	0.631	0.006
0	1.000	1.415	9.017	0.354	0.005	0.018	2.507	0.034	0.707	0.003	0.650	0.003
−3	1.413	1.731	11.911	0.499	0.007	0.021	2.788	0.044	0.578	0.005	0.658	0.003
−6	1.995	2.233	16.014	0.680	0.008	0.022	2.933	0.051	0.449	0.006	0.664	0.002

−9	2.818	2.991	21.950	0.825	0.009	0.021	2.990	0.055	0.335	0.007	0.674	0.004
−12	3.981	4.106	30.466	1.266	0.009	0.020	3.008	0.058	0.244	0.008	0.681	0.003

D) signal: white Gaussian random, noise: white Gaussian random

SNR	RMSE	RMS	Max-min		Skewness		Kurtosis		Corr.Coeff.		E-CDF	
(dB)	Mean	Mean	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
∞	0.000	1.000	6.951	0.000	0.009	0.000	2.915	0.000	1.000	0.000	0.000	0.000
12	0.251	1.032	7.355	0.199	0.009	0.011	2.917	0.018	0.970	0.000	0.005	0.003
9	0.355	1.062	7.595	0.254	0.009	0.015	2.923	0.024	0.943	0.000	0.006	0.004
6	0.501	1.120	8.076	0.336	0.010	0.020	2.932	0.031	0.894	0.001	0.008	0.008
3	0.708	1.227	8.980	0.448	0.011	0.025	2.947	0.042	0.817	0.002	0.011	0.014
0	1.000	1.416	10.572	0.586	0.012	0.028	2.963	0.056	0.708	0.004	0.017	0.023
−3	1.413	1.733	12.942	0.911	0.014	0.027	2.978	0.067	0.580	0.006	0.024	0.035
−6	1.995	2.235	16.699	1.121	0.015	0.025	2.990	0.074	0.450	0.008	0.030	0.048
−9	2.818	2.994	22.388	1.149	0.016	0.022	2.999	0.077	0.337	0.009	0.036	0.059
−12	3.981	4.108	30.767	1.161	0.016	0.020	3.005	0.077	0.247	0.010	0.042	0.068

Table 3.4: Statistical measures of the noisy test signals with ECG as the signal and white Gaussian random waveform as the noise. Mean & standard deviation (SD) for 10 noise records

A) signal: 105_00, noise: white Gaussian

SNR	RMSE	RMS	Max-min		Skewness		Kurtosis		Corr.Coeff.		E-CDF	
(dB)	Mean	Mean	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
∞	0.000	1.000	8.892	0.000	2.914	0.000	13.063	0.000	1.000	0.000	0.000	0.000
12	0.251	1.031	9.576	0.189	2.663	0.010	11.949	0.059	0.970	0.000	0.059	0.013
9	0.355	1.061	9.951	0.243	2.445	0.014	10.999	0.074	0.942	0.000	0.095	0.022
6	0.501	1.119	10.564	0.316	2.090	0.017	9.504	0.084	0.894	0.001	0.151	0.035
3	0.708	1.226	11.643	0.345	1.594	0.021	7.549	0.083	0.816	0.001	0.228	0.042
0	1.000	1.415	13.304	0.450	1.040	0.022	5.592	0.070	0.707	0.003	0.318	0.047
−3	1.413	1.731	15.746	0.564	0.571	0.020	4.178	0.052	0.578	0.004	0.404	0.051
−6	1.995	2.232	19.405	0.766	0.268	0.017	3.440	0.040	0.448	0.005	0.472	0.059
−9	2.818	2.991	25.209	0.752	0.112	0.015	3.145	0.035	0.335	0.006	0.524	0.066
−12	3.981	4.105	33.847	1.114	0.042	0.014	3.045	0.033	0.244	0.007	0.561	0.071

B) signal: 106_00, noise: white Gaussian

SNR	RMSE	RMS	Max-min		Skewness		Kurtosis		Corr.Coeff.		E-CDF	
(dB)	Mean	Mean	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
∞	0.000	1.000	10.124	0.000	3.504	0.000	20.712	0.000	1.000	0.000	0.000	0.000
12	0.251	1.031	10.544	0.206	3.197	0.012	18.664	0.085	0.970	0.000	0.049	0.024
9	0.355	1.061	10.845	0.313	2.934	0.015	16.967	0.103	0.942	0.000	0.087	0.039
6	0.501	1.119	11.463	0.355	2.506	0.017	14.315	0.114	0.894	0.001	0.147	0.048
3	0.708	1.226	12.495	0.402	1.908	0.018	10.870	0.118	0.816	0.002	0.226	0.056
0	1.000	1.415	14.130	0.492	1.244	0.019	7.448	0.117	0.708	0.003	0.313	0.051
−3	1.413	1.732	16.626	0.659	0.682	0.020	4.997	0.101	0.578	0.005	0.394	0.050
−6	1.995	2.233	20.315	0.913	0.321	0.018	3.734	0.073	0.449	0.007	0.462	0.049
−9	2.818	2.992	25.917	1.272	0.137	0.015	3.236	0.047	0.335	0.008	0.514	0.049
−12	3.981	4.106	34.070	1.825	0.056	0.013	3.073	0.031	0.245	0.009	0.553	0.047

C) signal: 107_00, noise: white Gaussian

SNR	RMSE	RMS	Max-min		Skewness		Kurtosis		Corr.Coeff.		E-CDF	
(dB)	Mean	Mean	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
∞	0.000	1.000	7.224	0.000	-0.297	0.000	3.982	0.000	1.000	0.000	0.000	0.000
12	0.251	1.032	7.876	0.172	-0.272	0.009	3.872	0.016	0.970	0.000	0.075	0.020
9	0.355	1.063	8.318	0.243	-0.251	0.011	3.780	0.019	0.943	0.000	0.106	0.020
6	0.501	1.121	8.980	0.323	-0.215	0.013	3.634	0.020	0.894	0.001	0.139	0.018
3	0.708	1.228	10.032	0.431	-0.165	0.014	3.443	0.018	0.817	0.001	0.171	0.017
0	1.000	1.417	11.703	0.552	-0.110	0.013	3.253	0.016	0.709	0.003	0.200	0.016
−3	1.413	1.734	14.203	0.717	-0.062	0.011	3.116	0.018	0.580	0.004	0.231	0.015
−6	1.995	2.236	18.374	0.834	-0.031	0.012	3.044	0.022	0.451	0.005	0.264	0.016
−9	2.818	2.995	24.613	1.053	-0.015	0.013	3.014	0.025	0.338	0.006	0.294	0.016
−12	3.981	4.109	33.716	1.492	-0.007	0.015	3.004	0.028	0.248	0.006	0.319	0.017

Table 3.5: Statistical measures of the noisy test signals with concatenated ECG as the signal and white Gaussian random waveform as the noise. Mean & standard deviation (SD) for 10 noise records.

A) signal: 105_00, noise: white Gaussian

SNR	RMSE	RMS	Max-min		Skewness		Kurtosis		Corr.Coeff.		E-CDF	
(dB)	Mean	Mean	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
∞	0.000	1.000	5.531	0.000	2.833	0.000	11.763	0.000	1.000	0.000	0.000	0.000
12	0.251	1.031	6.768	0.253	2.575	0.008	10.694	0.017	0.970	0.000	0.107	0.045
9	0.355	1.060	7.332	0.301	2.358	0.012	9.831	0.018	0.942	0.000	0.138	0.054
6	0.501	1.118	8.158	0.286	2.008	0.018	8.493	0.029	0.894	0.001	0.197	0.057
3	0.708	1.224	9.351	0.505	1.521	0.019	6.772	0.038	0.816	0.001	0.274	0.057
0	1.000	1.413	11.071	0.400	0.984	0.030	5.086	0.036	0.706	0.002	0.361	0.057
−3	1.413	1.729	13.534	0.760	0.533	0.014	3.906	0.053	0.577	0.002	0.438	0.060
−6	1.995	2.230	17.213	1.147	0.248	0.020	3.320	0.051	0.447	0.002	0.498	0.062
−9	2.818	2.989	22.615	1.820	0.106	0.022	3.103	0.071	0.333	0.002	0.545	0.066
−12	3.981	4.103	30.728	2.289	0.045	0.018	3.040	0.068	0.242	0.003	0.579	0.068

B) signal: 106_00, noise: white Gaussian

SNR	RMSE	RMS	Max-min		Skewness		Kurtosis		Corr.Coeff.		E-CDF	
(dB)	Mean	Mean	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
∞	0.000	1.000	7.485	0.000	2.080	0.000	8.735	0.000	1.000	0.000	0.000	0.000
12	0.251	1.192	8.909	0.257	1.890	0.005	8.035	0.017	0.970	0.000	0.024	0.026
9	0.355	1.277	9.668	0.213	1.743	0.009	7.547	0.023	0.942	0.000	0.038	0.033
6	0.501	1.402	10.499	0.433	1.486	0.011	6.683	0.032	0.894	0.001	0.065	0.042
3	0.708	1.583	11.845	0.622	1.145	0.021	5.641	0.026	0.816	0.001	0.114	0.050
0	1.000	1.849	13.848	0.804	0.733	0.011	4.365	0.027	0.706	0.001	0.182	0.054
−3	1.413	2.236	16.659	0.989	0.397	0.018	3.603	0.046	0.577	0.002	0.271	0.060
−6	1.995	2.796	20.482	0.918	0.182	0.015	3.201	0.049	0.446	0.003	0.377	0.077
−9	2.818	3.598	26.116	1.011	0.081	0.010	3.085	0.042	0.332	0.003	0.489	0.101
−12	3.981	4.741	35.477	2.308	0.029	0.028	3.054	0.045	0.241	0.004	0.589	0.123

C) signal: 107_00, noise: white Gaussian

SNR	RMSE	RMS	Max-min		Skewness		Kurtosis		Corr.Coeff.		E-CDF	
(dB)	Mean	Mean	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
∞	0.000	1.000	7.595	0.000	-0.004	0.000	3.076	0.000	1.000	0.000	0.000	0.000
12	0.251	1.192	9.252	0.185	-0.001	0.006	3.077	0.059	0.970	0.000	0.271	0.056
9	0.355	1.277	9.823	0.258	-0.004	0.011	3.060	0.074	0.942	0.000	0.332	0.059
6	0.501	1.401	10.795	0.244	-0.002	0.011	3.062	0.084	0.894	0.001	0.379	0.058
3	0.708	1.585	12.073	0.291	-0.011	0.016	3.058	0.083	0.816	0.001	0.413	0.056
0	1.000	1.850	14.351	0.622	0.005	0.016	3.061	0.070	0.707	0.002	0.438	0.051
−3	1.413	2.237	17.125	0.804	-0.003	0.008	3.070	0.052	0.577	0.002	0.459	0.051
−6	1.995	2.797	21.541	1.581	-0.013	0.014	3.063	0.040	0.448	0.002	0.476	0.049
−9	2.818	3.598	28.219	1.800	0.001	0.019	3.059	0.035	0.334	0.002	0.493	0.049
−12	3.981	4.745	36.182	2.450	0.017	0.033	3.021	0.033	0.243	0.004	0.506	0.047

Table 3.6: Statistical measures of the noisy tests signal with recorded artifact as the signal and white Gaussian random waveform as the noise. Mean & standard deviation (SD) for 10 noise records.

A) signal: Baseline wander bwm_00, noise: white Gaussian

SNR	RMSE	RMS	Max-min		Skewness		Kurtosis		Corr.Coeff.		E-CDF	
(dB)	Mean	Mean	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
∞	0.000	1.000	5.209	0.000	-0.520	0.000	3.030	0.000	1.000	0.000	0.000	0.000
12	0.251	1.032	6.271	0.138	-0.473	0.004	3.021	0.012	0.970	0.000	0.037	0.006
9	0.355	1.062	6.878	0.195	-0.433	0.005	3.016	0.016	0.943	0.000	0.050	0.005
6	0.501	1.119	7.781	0.287	-0.369	0.007	3.010	0.023	0.894	0.001	0.064	0.004
3	0.708	1.226	9.075	0.426	-0.281	0.008	3.003	0.030	0.817	0.001	0.080	0.004
0	1.000	1.416	10.959	0.603	-0.182	0.010	2.998	0.037	0.708	0.002	0.104	0.004
−3	1.413	1.732	13.721	0.797	-0.100	0.012	2.997	0.041	0.579	0.003	0.138	0.004
−6	1.995	2.234	17.921	0.939	-0.047	0.014	2.998	0.043	0.449	0.004	0.176	0.005
−9	2.818	2.992	24.183	1.239	-0.020	0.015	3.000	0.043	0.336	0.005	0.211	0.006
−12	3.981	4.107	33.195	1.624	-0.009	0.015	3.003	0.042	0.245	0.005	0.238	0.007

B) signal: EMG mam_00, noise: white Gaussian

SNR	RMSE	RMS	Max-min		Skewness		Kurtosis		Corr.Coeff.		E-CDF	
(dB)	Mean	Mean	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
∞	0.000	1.000	10.320	0.000	0.601	0.000	6.112	0.000	1.000	0.000	0.000	0.000
12	0.251	1.032	10.711	0.318	0.552	0.012	5.749	0.029	0.970	0.000	0.038	0.007
9	0.355	1.063	11.068	0.386	0.508	0.016	5.451	0.038	0.943	0.000	0.058	0.008
6	0.501	1.121	11.632	0.490	0.435	0.020	4.985	0.047	0.894	0.001	0.084	0.006
3	0.708	1.228	12.465	0.627	0.333	0.022	4.382	0.053	0.817	0.001	0.117	0.005
0	1.000	1.418	13.764	0.726	0.219	0.021	3.783	0.054	0.709	0.002	0.154	0.003
−3	1.413	1.735	15.848	0.832	0.121	0.018	3.355	0.051	0.581	0.004	0.194	0.004
−6	1.995	2.237	19.429	0.925	0.059	0.016	3.135	0.048	0.452	0.005	0.230	0.003
−9	2.818	2.996	25.265	0.948	0.026	0.015	3.050	0.047	0.339	0.005	0.261	0.004
−12	3.981	4.110	34.017	1.371	0.012	0.015	3.023	0.048	0.248	0.006	0.286	0.004

C) signal: motion artifact emm_00, noise: white Gaussian

SNR	RMSE	RMS	Max-min		Skewness		Kurtosis		Corr.Coeff.		E-CDF	
(dB)	Mean	Mean	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
∞	0.000	1.000	4.693	0.000	0.154	0.000	2.902	0.000	1.000	0.000	0.000	0.000
12	0.251	1.032	6.216	0.120	0.143	0.006	2.918	0.011	0.970	0.000	0.053	0.004
9	0.355	1.063	6.892	0.188	0.133	0.008	2.931	0.014	0.943	0.000	0.072	0.004
6	0.501	1.121	7.850	0.284	0.116	0.010	2.950	0.017	0.894	0.001	0.087	0.004
3	0.708	1.228	9.252	0.377	0.091	0.012	2.976	0.019	0.817	0.001	0.103	0.004
0	1.000	1.418	11.270	0.495	0.064	0.015	3.002	0.021	0.709	0.002	0.124	0.005
−3	1.413	1.735	14.166	0.665	0.040	0.017	3.021	0.023	0.581	0.004	0.154	0.005
−6	1.995	2.236	18.448	0.911	0.024	0.018	3.029	0.025	0.452	0.005	0.190	0.005
−9	2.818	2.995	24.814	1.188	0.015	0.018	3.031	0.027	0.339	0.006	0.224	0.006
−12	3.981	4.110	34.060	1.506	0.011	0.018	3.030	0.028	0.248	0.006	0.251	0.006

Table 3.7: Statistical measures of the noisy test signals with ECG as the signal and EMG as the noise.

A) signal: ECG-105_00, noise: EMG mam_00

SNR (dB)	RMSE	RMS	Max-min	Skewness	Kurtosis	Corr.Coeff.	E-CDF
∞	0.000	1.000	8.892	2.914	13.063	1.000	0.000
12	0.251	1.026	9.228	2.692	11.980	0.970	0.051
9	0.355	1.054	9.519	2.494	11.047	0.942	0.078
6	0.501	1.110	9.994	2.174	9.611	0.892	0.116
3	0.708	1.214	11.085	1.739	7.840	0.812	0.170
0	1.000	1.400	13.490	1.278	6.298	0.700	0.229
−3	1.413	1.714	17.158	0.920	5.511	0.567	0.284
−6	1.995	2.214	22.405	0.719	5.422	0.434	0.333
−9	2.818	2.972	30.213	0.636	5.621	0.317	0.373
−12	3.981	4.085	41.744	0.610	5.829	0.225	0.403

B) signal: ECG-106_00, noise: EMG mam_00

SNR (dB)	RMSE	RMS	Max-min	Skewness	Kurtosis	Corr.Coeff.	E-CDF
∞	0.000	1.000	10.124	3.504	20.712	1.000	0.000
12	0.251	1.026	10.675	3.206	18.798	0.970	0.055
9	0.355	1.055	11.166	2.951	17.128	0.942	0.075
6	0.501	1.110	12.102	2.542	14.527	0.892	0.107
3	0.708	1.214	13.728	1.989	11.240	0.812	0.143
0	1.000	1.400	16.025	1.406	8.207	0.700	0.186
−3	1.413	1.715	19.470	0.954	6.383	0.567	0.236
−6	1.995	2.214	24.520	0.705	5.780	0.434	0.293
−9	2.818	2.972	31.655	0.606	5.784	0.318	0.348
−12	3.981	4.086	41.732	0.581	5.923	0.226	0.385

C) signal: ECG-107_00, noise: EMG mam_00

SNR (dB)	RMSE	RMS	Max-min	Skewness	Kurtosis	Corr.Coeff.	E-CDF
∞	0.000	1.000	7.224	-0.297	3.982	1.000	0.000
12	0.251	1.035	8.072	-0.232	3.891	0.970	0.038
9	0.355	1.067	8.641	-0.191	3.834	0.943	0.064
6	0.501	1.126	9.453	-0.121	3.786	0.895	0.099
3	0.708	1.235	10.912	-0.015	3.827	0.819	0.133
0	1.000	1.426	13.202	0.123	4.065	0.713	0.165
−3	1.413	1.744	16.713	0.268	4.514	0.587	0.200
−6	1.995	2.246	22.493	0.389	5.030	0.460	0.225
−9	2.818	3.006	30.657	0.473	5.456	0.348	0.255
−12	3.981	4.121	42.337	0.526	5.741	0.259	0.288

Table 3.8 Statistical measures of the noisy test signal with ECG as the signal and MA as the noise.

A) signal: ECG-105_00, noise: motion artifact emm_00

SNR (dB)	RMSE	RMS	Max-min	Skewness	Kurtosis	Corr.Coeff.	E-CDF
∞	0.000	1.000	8.892	2.914	13.063	1.000	0.000
12	0.251	1.028	9.727	2.718	12.262	0.970	0.086
9	0.355	1.057	10.177	2.525	11.433	0.942	0.119
6	0.501	1.112	10.814	2.200	10.064	0.893	0.152
3	0.708	1.217	11.714	1.735	8.186	0.814	0.193
0	1.000	1.405	12.984	1.205	6.198	0.702	0.241
−3	1.413	1.720	14.792	0.745	4.645	0.570	0.282
−6	1.995	2.220	17.354	0.437	3.726	0.438	0.312
−9	2.818	2.978	21.284	0.269	3.274	0.323	0.332
−12	3.981	4.092	26.998	0.184	3.065	0.231	0.351

B) signal: ECG-106_00, noise: motion artifact emm_00

SNR (dB)	RMSE	RMS	Max-min	Skewness	Kurtosis	Corr.Coeff.	E-CDF
∞	0.000	1.000	10.124	3.504	20.712	1.000	0.000
12	0.251	1.022	10.569	3.276	19.093	0.969	0.060
9	0.355	1.048	10.906	3.026	17.475	0.941	0.084
6	0.501	1.102	11.383	2.598	14.826	0.891	0.122
3	0.708	1.203	12.160	1.980	11.258	0.809	0.166
0	1.000	1.387	13.444	1.279	7.626	0.694	0.210
−3	1.413	1.700	15.290	0.690	4.999	0.557	0.251
−6	1.995	2.198	17.896	0.326	3.644	0.421	0.285
−9	2.818	2.955	21.578	0.153	3.109	0.303	0.309
−12	3.981	4.068	26.779	0.087	2.930	0.209	0.326

C) signal: ECG-107_00, noise: motion artifact emm_00

SNR (dB)	RMSE	RMS	Max-min	Skewness	Kurtosis	Corr.Coeff.	E-CDF
∞	0.000	1.000	7.224	-0.297	3.982	1.000	0.000
12	0.251	1.020	7.642	-0.291	3.928	0.969	0.059
9	0.355	1.045	7.944	-0.276	3.846	0.941	0.085
6	0.501	1.097	8.385	-0.248	3.700	0.890	0.117
3	0.708	1.198	9.028	-0.202	3.486	0.807	0.140
0	1.000	1.381	9.985	-0.139	3.248	0.690	0.156
−3	1.413	1.692	11.425	-0.074	3.054	0.552	0.184
−6	1.995	2.189	13.986	-0.021	2.938	0.414	0.214
−9	2.818	2.946	18.828	0.016	2.881	0.294	0.239
−12	3.981	4.059	25.674	0.039	2.857	0.200	0.259

Table 3.9 Statistical measures of the noisy test signal with ECG as the signal and BW as the noise

A) signal: ECG-105_00, noise: baseline wander bwm_00

SNR (dB)	RMSE	RMS	Max-min	Skewness	Kurtosis	Corr.Coeff.	E-CDF
∞	0.000	1.000	8.892	2.914	13.063	1.000	0.000
12	0.251	1.031	9.066	2.681	12.109	0.970	0.066
9	0.355	1.061	9.326	2.456	11.200	0.942	0.087
6	0.501	1.119	9.911	2.074	9.733	0.894	0.128
3	0.708	1.225	10.738	1.517	7.773	0.816	0.176
0	1.000	1.414	11.905	0.867	5.772	0.707	0.225
−3	1.413	1.731	13.554	0.284	4.298	0.578	0.266
−6	1.995	2.232	15.884	-0.117	3.512	0.448	0.298
−9	2.818	2.991	19.364	-0.339	3.190	0.334	0.318
−12	3.981	4.105	24.518	-0.446	3.079	0.244	0.334

B) signal: ECG-106_00, noise: baseline wander bwm_00

SNR (dB)	RMSE	RMS	Max-min	Skewness	Kurtosis	Corr.Coeff.	E-CDF
∞	0.000	1.000	10.124	3.504	20.712	1.000	0.000
12	0.251	1.043	10.986	3.031	18.112	0.971	0.058
9	0.355	1.077	11.374	2.707	16.375	0.944	0.091
6	0.501	1.140	11.939	2.207	13.824	0.898	0.145
3	0.708	1.253	12.848	1.536	10.664	0.826	0.196
0	1.000	1.448	14.132	0.804	7.601	0.724	0.241
−3	1.413	1.770	15.946	0.187	5.383	0.604	0.275
−6	1.995	2.275	18.661	-0.214	4.160	0.482	0.298
−9	2.818	3.036	22.619	-0.420	3.597	0.375	0.316
−12	3.981	4.152	28.211	-0.508	3.346	0.288	0.332

C) signal: ECG-107_00, noise: baseline wander bwm_00

SNR (dB)	RMSE	RMS	Max-min	Skewness	Kurtosis	Corr.Coeff.	E-CDF
∞	0.000	1.000	7.224	-0.297	3.982	1.000	0.000
12	0.251	1.025	7.891	-0.284	3.892	0.970	0.068
9	0.355	1.053	8.316	-0.282	3.823	0.942	0.097
6	0.501	1.108	8.915	-0.289	3.721	0.892	0.120
3	0.708	1.211	9.761	-0.315	3.595	0.812	0.138
0	1.000	1.397	10.963	-0.366	3.472	0.699	0.157
−3	1.413	1.711	12.662	-0.431	3.371	0.565	0.183
−6	1.995	2.211	15.364	-0.488	3.291	0.431	0.209
−9	2.818	2.968	19.313	-0.524	3.225	0.314	0.234
−12	3.981	4.082	24.89	-0.544	3.172	0.222	0.255

Table 3.10: Denoising performance indices calculated for noisy waveforms with different SNRs. Mean and s.d. for 15 ECG records.

a) White Gaussian random

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
β	1.000	0.002	1.000	0.005	0.998	0.007	1.004	0.014	1.008	0.023
γ	0.998	0.034	0.997	0.018	0.998	0.008	1.002	0.005	1.000	0.003
κ	0.059	0.018	0.078	0.037	0.099	0.042	0.133	0.069	0.204	0.045

b) BW

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
β	1.002	0.009	1.004	0.019	1.009	0.037	1.017	0.074	1.034	0.147
γ	1.034	0.147	1.017	0.074	1.009	0.037	1.004	0.019	1.002	0.009
κ	0.110	0.056	0.154	0.078	0.218	0.110	0.308	0.156	0.437	0.222

c) EMG noise

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
β	0.995	0.007	0.990	0.015	0.980	0.030	0.961	0.059	0.922	0.118
γ	0.922	0.118	0.961	0.059	0.980	0.030	0.990	0.015	0.995	0.007
κ	0.097	0.048	0.140	0.071	0.198	0.102	0.279	0.142	0.388	0.192

d) MA

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
Corr.Coeff.	0.999	0.009	0.998	0.017	0.997	0.035	0.993	0.069	0.987	0.138
γ	0.987	0.138	0.993	0.069	0.997	0.035	0.998	0.017	0.999	0.009
κ	0.116	0.037	0.164	0.051	0.232	0.072	0.327	0.102	0.462	0.146

Table 3.11: Insertion and detection errors as a function of SNR for noisy ECG using the initial 1-minute segments of the records ECG-105, ECG-106, and ECG-112 as noise-free signals added with white Gaussian noise.

a) ECG record: 105 (No. of R-peaks = 82)

Temporal Tolerance (ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.561	0.646	0.744	1.439	1.549	0.561	0.634	0.707	0.915	0.927
10	0.000	0.012	0.049	0.720	1.207	0.000	0.000	0.012	0.195	0.585
20	0.000	0.012	0.037	0.646	1.098	0.000	0.000	0.000	0.122	0.476
30	0.000	0.012	0.037	0.598	1.085	0.000	0.000	0.000	0.073	0.463
40	0.000	0.012	0.037	0.573	1.049	0.000	0.000	0.000	0.049	0.427
50	0.000	0.012	0.037	0.537	1.049	0.000	0.000	0.000	0.012	0.427
60	0.000	0.012	0.037	0.537	1.000	0.000	0.000	0.000	0.012	0.378
70	0.000	0.012	0.037	0.537	0.976	0.000	0.000	0.000	0.012	0.354
80	0.000	0.012	0.037	0.537	0.963	0.000	0.000	0.000	0.012	0.341
90	0.000	0.012	0.037	0.537	0.939	0.000	0.000	0.000	0.012	0.317
100	0.000	0.012	0.037	0.524	0.915	0.000	0.000	0.000	0.012	0.293

b) ECG record: 106 (No. of R-peaks = 61)

Temporal Tolerance (ms)	Insertion Error					Detection Error _				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.689	0.803	0.754	1.820	1.148	0.689	0.738	0.902	0.934	0.967
10	0.066	0.131	0.082	1.393	0.951	0.066	0.066	0.230	0.508	0.770
20	0.049	0.082	0.049	1.377	0.885	0.049	0.016	0.197	0.492	0.705
30	0.049	0.082	0.049	1.361	0.852	0.049	0.016	0.197	0.475	0.672
40	0.033	0.082	0.049	1.311	0.836	0.033	0.016	0.197	0.426	0.656
50	0.033	0.082	0.049	1.262	0.820	0.033	0.016	0.197	0.377	0.639
60	0.016	0.082	0.033	1.246	0.803	0.016	0.016	0.180	0.361	0.623
70	0.016	0.082	0.016	1.246	0.787	0.016	0.016	0.164	0.361	0.607
80	0.016	0.082	0.016	1.213	0.787	0.016	0.016	0.164	0.328	0.607
90	0.016	0.082	0.016	1.213	0.770	0.016	0.016	0.164	0.328	0.590
100	0.016	0.082	0.016	1.164	0.754	0.016	0.016	0.164	0.279	0.574

c) ECG record: 112 (No. of R-peaks = 85)

Temporal Tolerance (ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.529	0.600	0.835	1.424	1.306	0.529	0.600	0.706	0.871	0.965
10	0.000	0.000	0.165	0.812	1.071	0.000	0.000	0.035	0.259	0.729
20	0.000	0.000	0.129	0.741	0.965	0.000	0.000	0.000	0.188	0.624
30	0.000	0.000	0.129	0.741	0.929	0.000	0.000	0.000	0.188	0.588
40	0.000	0.000	0.129	0.729	0.929	0.000	0.000	0.000	0.176	0.588
50	0.000	0.000	0.129	0.729	0.882	0.000	0.000	0.000	0.176	0.541
60	0.000	0.000	0.129	0.706	0.824	0.000	0.000	0.000	0.153	0.494
70	0.000	0.000	0.129	0.659	0.753	0.000	0.000	0.000	0.106	0.424
80	0.000	0.000	0.129	0.635	0.729	0.000	0.000	0.000	0.082	0.400

90	0.000	0.000	0.129	0.624	0.706	0.000	0.000	0.000	0.071	0.376
100	0.000	0.000	0.129	0.600	0.682	0.000	0.000	0.000	0.047	0.353

Table 3.12: Insertion and detection errors as a function of SNR for noisy ECG generated using the initial 1-minute segments of the records ECG-105, ECG-106, and ECG-112 as noise-free signals added with baseline wander as noise.

a) ECG record: 105 (No. of R-peaks = 82)

Temporal Tolerance (ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.012	0.012	0.073	0.183	0.329	0.012	0.012	0.073	0.171	0.317
10	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
30	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
40	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
50	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
60	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
70	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
80	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
90	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
100	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000

b) ECG record: 106 (No. of R-peaks = 61)

Temporal Tolerance (ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.213	0.213	0.279	0.295	0.180	0.213	0.213	0.328	0.541	0.738
10	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.066	0.262	0.574
20	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.066	0.262	0.574
30	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.066	0.262	0.574
40	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.066	0.262	0.574
50	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.066	0.262	0.574
60	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.066	0.262	0.574
70	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.066	0.262	0.574
80	0.000	0.000	0.000	0.016	0.016	0.000	0.000	0.049	0.262	0.574
90	0.000	0.000	0.000	0.016	0.016	0.000	0.000	0.049	0.262	0.574
100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.049	0.246	0.55

c) ECG record: 112 (No. of R-peaks = 85)

Temporal Tolerance (ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.071	0.071	0.082	0.200	0.318	0.071	0.071	0.082	0.200	0.318
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
30	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
40	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
60	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
70	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
80	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
90	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 3.13: Insertion and detection errors as a function of SNR for noisy ECG generated using the initial 1-minute segments of the records ECG-105, ECG-106, and ECG-112 as noise-free signals added with motion artifact as noise.

a) ECG record: 105 (No. of R-peaks = 82)

Temporal Tolerance (ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.012	0.049	0.085	0.390	0.463	0.012	0.049	0.085	0.268	0.939
10	0.000	0.000	0.000	0.122	0.354	0.000	0.000	0.000	0.000	0.829
20	0.000	0.000	0.000	0.122	0.317	0.000	0.000	0.000	0.000	0.793
30	0.000	0.000	0.000	0.122	0.305	0.000	0.000	0.000	0.000	0.780
40	0.000	0.000	0.000	0.122	0.305	0.000	0.000	0.000	0.000	0.780
50	0.000	0.000	0.000	0.122	0.305	0.000	0.000	0.000	0.000	0.780
60	0.000	0.000	0.000	0.122	0.305	0.000	0.000	0.000	0.000	0.780
70	0.000	0.000	0.000	0.122	0.293	0.000	0.000	0.000	0.000	0.768
80	0.000	0.000	0.000	0.122	0.280	0.000	0.000	0.000	0.000	0.756
90	0.000	0.000	0.000	0.122	0.256	0.000	0.000	0.000	0.000	0.732
100	0.000	0.000	0.000	0.122	0.256	0.000	0.000	0.000	0.000	0.732

b) ECG record: 106 (No. of R-peaks = 61)

Temporal Tolerance (ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.090	0.134	0.164	0.394	0.448	0.090	0.134	0.284	0.507	0.892
10	0.033	0.033	0.033	0.295	0.328	0.016	0.033	0.115	0.377	0.885
20	0.033	0.033	0.033	0.295	0.328	0.016	0.033	0.115	0.377	0.885
30	0.033	0.033	0.033	0.295	0.328	0.016	0.033	0.115	0.377	0.885
40	0.033	0.033	0.033	0.279	0.311	0.016	0.033	0.115	0.361	0.869
50	0.033	0.033	0.033	0.279	0.311	0.016	0.033	0.115	0.361	0.869
60	0.033	0.033	0.033	0.262	0.311	0.016	0.033	0.115	0.344	0.869
70	0.033	0.033	0.033	0.262	0.311	0.016	0.033	0.115	0.344	0.869
80	0.016	0.016	0.016	0.230	0.279	0.000	0.016	0.098	0.311	0.836
90	0.016	0.016	0.016	0.230	0.279	0.000	0.016	0.098	0.311	0.836
100	0.016	0.016	0.016	0.230	0.279	0.000	0.016	0.098	0.311	0.836

c) ECG record: 112 (No. of R-peaks = 85)

Temporal Tolerance (ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.059	0.082	0.153	0.376	0.365	0.059	0.082	0.153	0.259	0.929
10	0.000	0.000	0.000	0.129	0.294	0.000	0.000	0.000	0.012	0.859
20	0.000	0.000	0.000	0.118	0.282	0.000	0.000	0.000	0.000	0.847
30	0.000	0.000	0.000	0.118	0.282	0.000	0.000	0.000	0.000	0.847
40	0.000	0.000	0.000	0.118	0.271	0.000	0.000	0.000	0.000	0.835
50	0.000	0.000	0.000	0.118	0.271	0.000	0.000	0.000	0.000	0.835
60	0.000	0.000	0.000	0.118	0.271	0.000	0.000	0.000	0.000	0.835
70	0.000	0.000	0.000	0.118	0.247	0.000	0.000	0.000	0.000	0.812
80	0.000	0.000	0.000	0.118	0.224	0.000	0.000	0.000	0.000	0.788
90	0.000	0.000	0.000	0.118	0.200	0.000	0.000	0.000	0.000	0.765
100	0.000	0.000	0.000	0.118	0.200	0.000	0.000	0.000	0.000	0.765

Table 3.14: Insertion and detection errors as a function of SNR for noisy ECG generated using the initial 1-minute segments of the records ECG105, ECG-106, and ECG112 as noise-free signals added with EMG artifact as noise.

a) ECG record: 105 (No. of R-peaks = 82)

Temporal Tolerance (ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.073	0.305	0.805	0.585	0.159	0.073	0.268	0.439	0.939	1.000
10	0.000	0.037	0.390	0.427	0.146	0.000	0.000	0.024	0.780	0.988
20	0.000	0.037	0.366	0.366	0.134	0.000	0.000	0.000	0.720	0.976
30	0.000	0.037	0.366	0.354	0.134	0.000	0.000	0.000	0.707	0.976
40	0.000	0.037	0.366	0.354	0.134	0.000	0.000	0.000	0.707	0.976
50	0.000	0.037	0.366	0.341	0.134	0.000	0.000	0.000	0.695	0.976
60	0.000	0.037	0.366	0.341	0.134	0.000	0.000	0.000	0.695	0.976
70	0.000	0.037	0.366	0.341	0.134	0.000	0.000	0.000	0.695	0.976
80	0.000	0.037	0.366	0.341	0.134	0.000	0.000	0.000	0.695	0.976
90	0.000	0.037	0.366	0.317	0.122	0.000	0.000	0.000	0.683	0.963
100	0.000	0.037	0.366	0.305	0.122	0.000	0.000	0.000	0.671	0.963

b) ECG record: 106 (No. of R-peaks = 61)

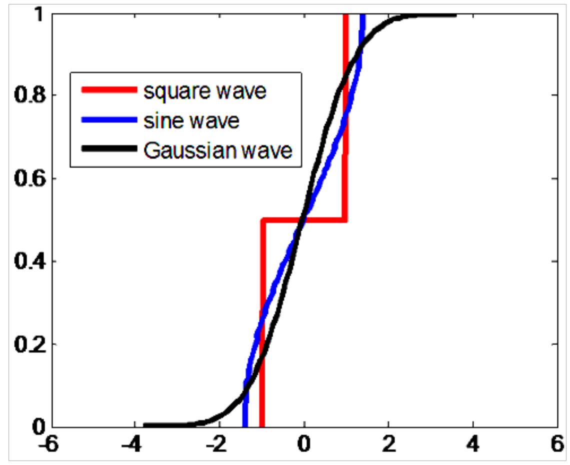
Temporal Tolerance (ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.344	0.459	0.787	0.656	0.115	0.328	0.508	0.705	0.967	1.000
10	0.033	0.066	0.475	0.557	0.115	0.016	0.115	0.393	0.869	1.000
20	0.033	0.066	0.426	0.557	0.115	0.016	0.115	0.344	0.869	1.000
30	0.033	0.049	0.410	0.525	0.115	0.016	0.098	0.328	0.836	1.000
40	0.033	0.049	0.410	0.525	0.115	0.016	0.098	0.328	0.836	1.000

50	0.033	0.049	0.410	0.525	0.115	0.016	0.098	0.328	0.836	1.000
60	0.033	0.049	0.410	0.508	0.082	0.016	0.098	0.328	0.836	0.967
70	0.033	0.049	0.410	0.508	0.082	0.016	0.098	0.328	0.836	0.967
80	0.033	0.049	0.410	0.492	0.066	0.016	0.098	0.328	0.820	0.951
90	0.033	0.049	0.410	0.492	0.066	0.016	0.098	0.328	0.820	0.951
100	0.016	0.033	0.393	0.475	0.066	0.000	0.082	0.311	0.803	0.951

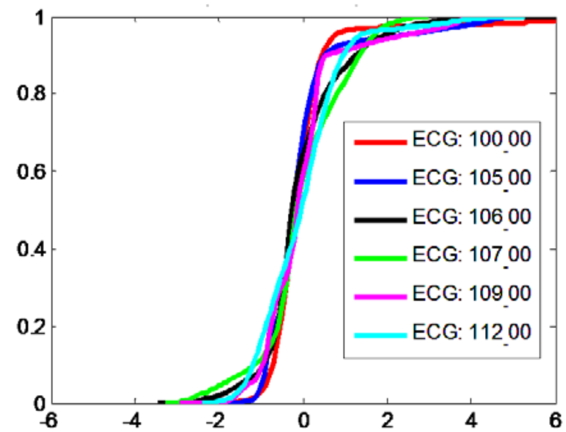
c) ECG record: 112 (No. of R-peaks = 85)

Temporal Tolerance (ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.118	0.365	0.659	0.635	0.153	0.106	0.224	0.365	0.894	1.000
10	0.012	0.141	0.318	0.400	0.141	0.000	0.000	0.024	0.659	0.988
20	0.012	0.141	0.318	0.388	0.141	0.000	0.000	0.024	0.647	0.988
30	0.012	0.141	0.318	0.388	0.141	0.000	0.000	0.024	0.647	0.988
40	0.012	0.141	0.318	0.376	0.118	0.000	0.000	0.024	0.635	0.965
50	0.012	0.141	0.318	0.376	0.118	0.000	0.000	0.024	0.635	0.965
60	0.012	0.141	0.318	0.365	0.106	0.000	0.000	0.024	0.624	0.953
70	0.012	0.141	0.318	0.365	0.106	0.000	0.000	0.024	0.624	0.953
80	0.012	0.141	0.318	0.341	0.094	0.000	0.000	0.024	0.600	0.941
90	0.012	0.141	0.318	0.341	0.094	0.000	0.000	0.024	0.600	0.941
100	0.012	0.141	0.318	0.341	0.094	0.000	0.000	0.024	0.600	0.941

A) Synthesized waveforms (sine wave, square wave, white Gaussian random)



B) ECG:(1-minute noise-free initial segments of the records 100, 105, 106, 107, 109, and 112 from MIT-BIH "mitdb" database)



C) Artifacts (1-minute initial segments of the records bwm, mam and emm from MIT-BIH "nstdb" database)

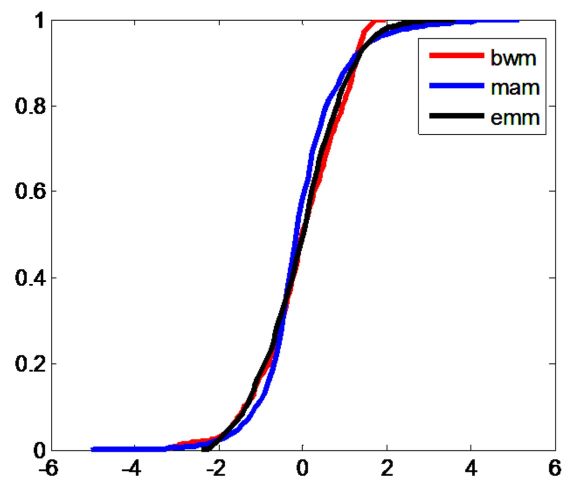


Figure 3.1: CDF for different test waveforms, with RMS of one: A) Synthesized waveforms (sine wave, square wave, and white Gaussian random), B) ECG:(1 minute noise-free initial segments of the records 100, 105, 106, 107, 109, and 112 from MIT-BIH "mitdb" database), C) Artifacts (1 minute initial segments of the records bwm, mam and emm from MIT-BIH "nstdb" database)

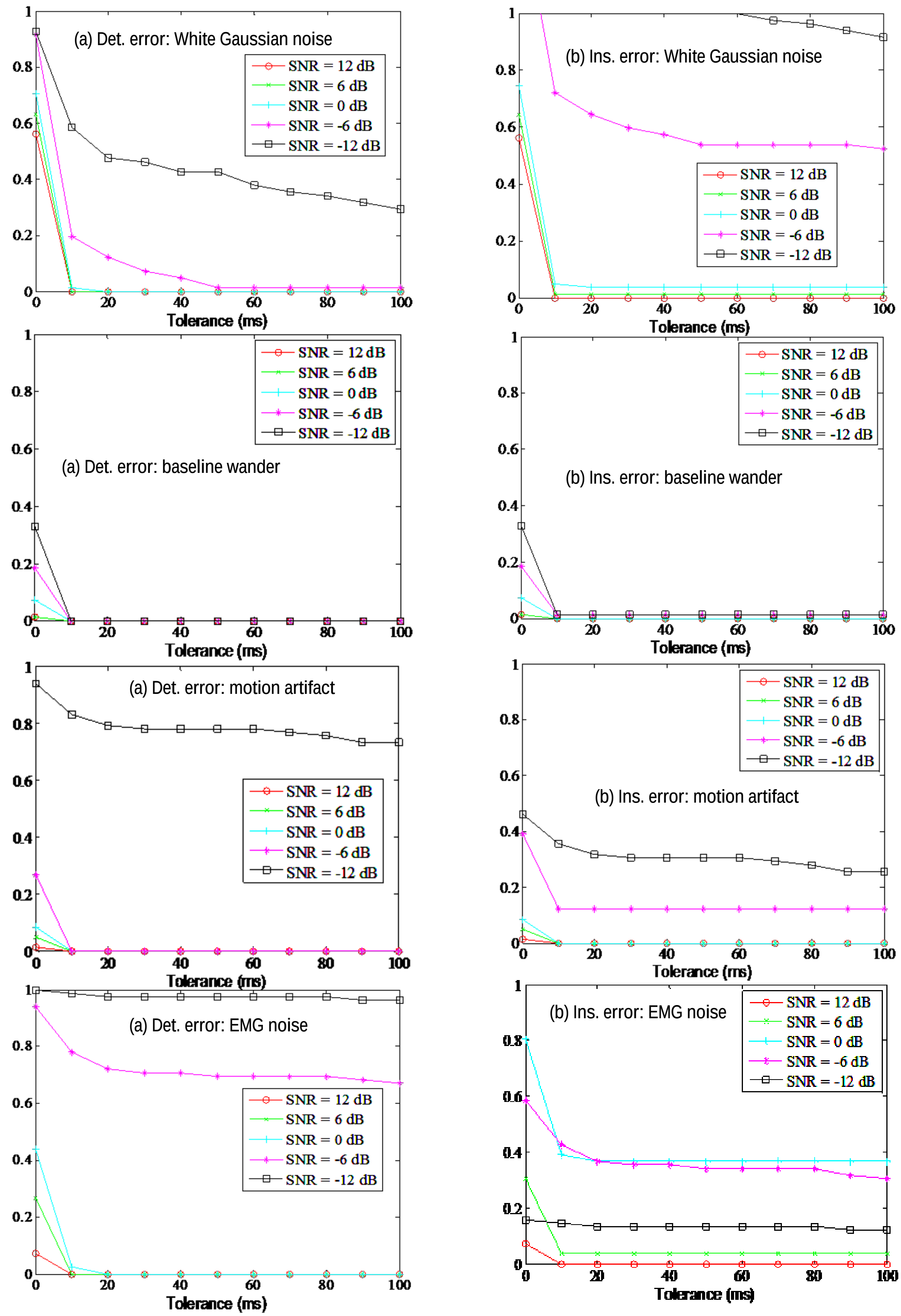


Figure 3.2: R-peak detection and insertion errors for noisy ECG signal, generated using ECG-105_00 and artifacts at different SNRs: (a) white Gaussian noise, (b) baseline wander artifact, (c) motion artifact, (d) EMG noise..

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Chapter 4

ARTIFACT SUPPRESSION USING THRESHOLDING AND LIMITING OF WAVELET COEFFICIENTS

4.1 Introduction

In ambulatory ECG recordings, the signal may be corrupted by BW, MA, and EMG noise. On the basis of a review of the ECG denoising literature in the second chapter, it has been concluded that wavelet-based methods are well suited for denoising of such recordings. The present work is a continuation of the investigations carried out in our lab on developing wavelet-based denoising techniques [16]-[18]. The main objective is to carry out a detailed assessment of the denoising technique based on thresholding and limiting of wavelet coefficients. As the effectiveness of the denoising is dependent on the choice of thresholds used for thresholding and limiting of wavelet coefficients, a detailed evaluation of the effect of the parameters used in threshold selection is to be carried out, using the methods as described in the third chapter.

Mithun [16] applied scale-dependent thresholding for ECG denoising using 9-level wavelet-decomposition of signals sampled at 360 Hz and investigated use of several mother wavelets for this application: 'db8', 'sym 5', 'sym 10', 'dmey', 'bior 6.8'. It was observed that details at scales 1, 8, and 9 contained dominantly noise and relatively little signal and the details at scales 3, 4, and 5 contained mostly signal and relatively little noise. Therefore, the signal was reconstructed from the details at the levels $3 - 7$. Tests indicated that the method was able to suppress baseline wander almost completely and EMG noise to some extent. The wavelets 'dmey' and 'symlet-10' were found to be most suitable for this application. It was observed that EMG and MA had significant contributions to the details at middle scales and therefore scale-dependent thresholding was not effective in suppressing them. Further investigations by Mithun *et al* [4] and Sebastian [17] showed that thresholding and limiting of the wavelet coefficients, using 'dmey' based wavelet analysis, was effective in suppressing EMG noise and MA, respectively.

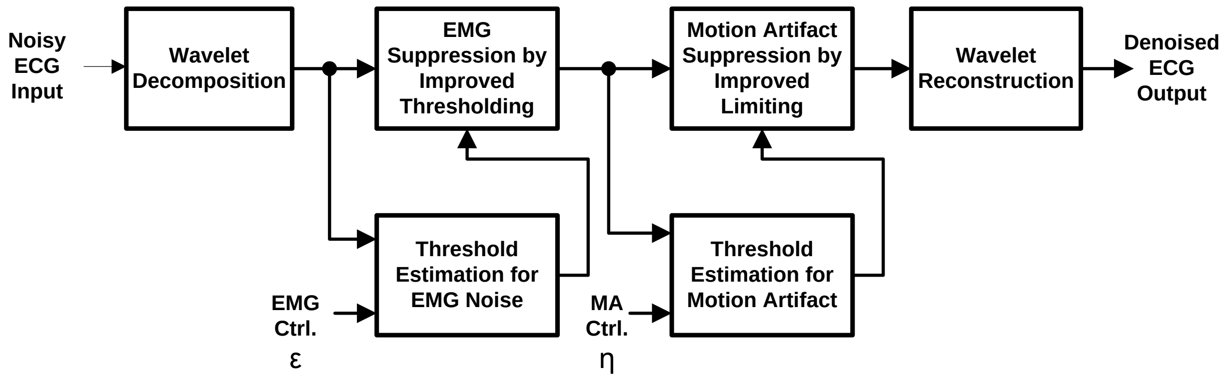

Figure 4.1: DWT-based ECG denoising as used by Pranava [11].

An improved thresholding function based on an exponential function was devised to combine the advantages of soft and hard thresholding functions for thresholding of the wavelet coefficients in order to suppress the EMG noise. A soft-limiting function based on a sinusoidal function was devised for limiting the wavelet coefficients in order to suppress MA. The thresholds for each scale were obtained as empirically selected quantiles of the cumulative distribution functions of the magnitude of the wavelet coefficients and scaled in accordance with selected control parameters, one for EMG and one for MA. Application of the technique on noisy ECG, generated by adding noise-free ECG and ECG-free artifacts, showed that it was effective in suppression of EMG noise and MA as indicated by $SNR_{impr,}$ improvement indices, and detection of R-peaks. Visual examination showed that the denoising method introduced pseudo-Gibbs oscillations. The oscillations were at a relatively low level and did not affect R-peak detection but could interfere with the detection of the weaker features in ECG. These oscillations could be suppressed by applying the denoising technique using translation-invariant wavelet transform (TIWT).

Pranava [18] used the technique of Mithun et al [4] to develop a LabWindows based GUI application, with the option of manually selecting the two control variables. A thresholding function based on a sinusoidal function was devised and was found to be comparable to that in [10]. To minimize pseudo-Gibbs oscillation resulting in the denoising process, use of TIWT and stationary wavelet transform (SWT) was investigated. It was reported that both were able to reduce the oscillations significantly, with TIWT performing slightly better.

For the present investigation, the basic approach of the denoising technique is the same used by Pranava [18], with a few modifications in the threshold determination process.

Suppression of BW is achieved by zeroing the approximation coefficients, EMG noise is suppressed by level-dependent thresholding of detail coefficients, and MA is suppressed by level-dependent limiting of detail coefficients. Thresholds for both of these operations are estimated using signal statistics. Matlab is used for the implementation of the denoising technique, generation of the test waveforms, and calculation of the performance indices. The denoising technique is described in the next section. A description of the various programs used in the investigation are given in Appendix A. Sample test results along with observations based on visual inspection of the waveforms are presented in Section 4.3. The results of the investigations on effect of the denoising parameters are presented in the following chapter.

4.2 Denoising technique for suppression of BW, EMG, and MA

Implementation of the noise suppression technique is shown in Figure 4.1. For sampling frequency of 360 Hz, 8-level decomposition is applied to obtain the detail coefficients $D_1(n)$ – $D_8(n)$, and the approximation coefficients $A_8(n)$. To suppress BW, the approximations coefficients are made zero. EMG noise and motion artifact are suppressed as described in the following subsections.

4.2.1 Suppression of EMG noise

EMG noise is due to voluntary and involuntary muscle activities in the body. In ambulatory recordings, EMG noise is always present and makes relatively low-level contribution to the magnitude of the detail coefficients, and hence thresholding can be used to suppress it [16]-[18]. The processing is carried out using two steps: (i) estimation of the thresholds from the statistics of the detail coefficients and (ii) thresholding the coefficients. For recordings with a sampling frequency of 360 Hz, EMG noise is found mainly in the scales 1 – 5 and hence thresholding is applied on these scales.

Time-varying threshold for each scale is calculated as a product of a level-dependent value obtained from the distribution of coefficient magnitudes, a time-varying scaling factor $\nu(n)$ related to the level of EMG noise, and a constant EMG control parameter ε with the range of (0 – 1). Thus, the time-varying threshold $\theta_j(n)$ for scale j is calculated as

$$\theta_j(n) = \varepsilon\, \nu(n)\ \mathrm{p90}\left(\left|D_j(i)\right|\right) \tag{4.1}$$

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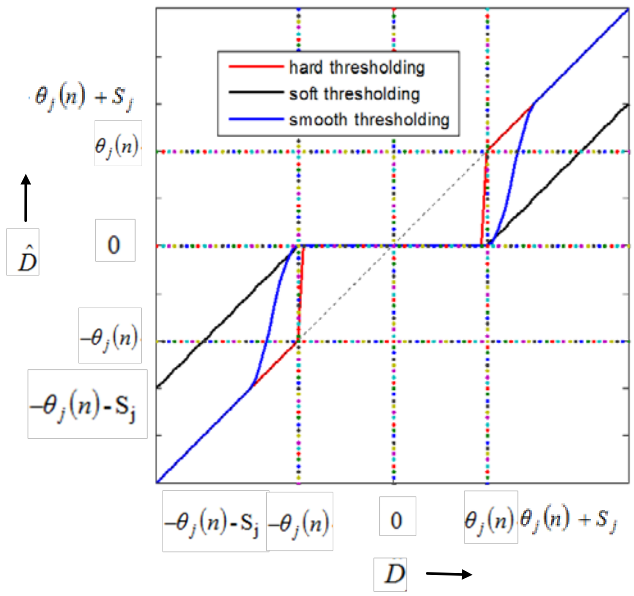

Figure 4.2: Smooth thresholding as described in the Equation 4.3 along with hard and soft thresholding

where $p90\lfloor D_j(i)\rfloor$ is the 90th percentile of the magnitude of the detail coefficients of the scale j. Level of EMG noise suppression can be controlled by the externally provided EMG control parameter ε. As the detail coefficients at level 1 represent almost negligible signal and contain major contribution from the noise, these are used for estimating the time-varying scaling factor $\nu(n)$. A typical EMG burst is generally longer in duration as compared to QRS. Hence, a 35-point moving average is applied on $|D_1(n)|$ to get $D_{1mLP}(n)$ as a time-varying estimate of noise magnitude and to suppress the possible contribution of QRS complexes. Its 5th and the 95th percentiles are used as lower and upper thresholds to get a normalized time-varying magnitude to serve as the scaling factor $\nu(n)$. It is given as the following:

$$\nu(n)=\begin{cases}0, & D_{1mLP}(n)<\mathrm{p5}(D_{1mLP}(i))\\ \dfrac{D_{1mLP}(n)-\mathrm{p5}(D_{1mLP}(i))}{\mathrm{p95}(D_{1mLP}(i))-\mathrm{p5}(D_{1mLP}(i))}, & \mathrm{p5}(D_{1mLP}(i))\le D_{1mLP}(n)\le \mathrm{p95}(D_{1mLP}(i))\\ 1, & D_{1mLP}(n)>\mathrm{p95}(D_{1mLP}(i))\end{cases} \tag{4.2}$$

The factor $\nu(n)$ is resampled for each scale for calculating the threshold $\theta_j(n)$ in accordance with Equation 4.1.

For the scales 2 – 5, the output coefficients $\hat{D}_j(n)$ are obtained using the following smooth thresholding function

$$\hat{D}_j(n) = \begin{cases} 0, & |D_j(n)| < \theta_j(n) \\ \frac{D_j(n)}{2}\left[1 - \cos\left(\frac{(|D_j(n)| - \theta_j(n))\pi}{S_i}\right)\right], & \theta_j(n) \le |D_j(n)| \le -\theta_j(n) - S_j \\ D_j(n), & |D_j(n)| > \theta_j(n) + S_j \end{cases} \tag{4.3}$$

which employs a sine-based smooth transition from soft thresholding to hard thresholding with the transition span S_j. The function is shown in Figure 4.2. In order to suppress EMG noise without introducing signal distortion, the span is selected as the 95th percentile of the supra-threshold coefficient values, i.e. given as

$$S_j = \mathrm{p95}(|(D_j(i))| - \theta_j(i), \ \forall \ |D_j(i)| > \theta_j(i)) \tag{4.4}$$

The detail coefficients of the scale 1 are set to zero while those of the scales 6 – 8 remain unmodified: $\hat{D}_1(n) = 0$, $\hat{D}_6(n) = D_6(n)$, $\hat{D}_7(n) = D_7(n)$, $\hat{D}_8(n) = D_8(n)$.

4.2.2 Suppression of motion artifact

During thresholding for suppression of EMG noise, it is assumed that the noise makes relatively low-magnitude contribution to the detail coefficients of the lower scales. In case of MA corruption, the artifact makes intermittent but relatively high-magnitude contribution to the detail coefficients of the higher scales. To suppress motion artifact, limiting of wavelet coefficient is used, with limiting threshold obtained from the statistics of the detail coefficients. MA suppression is carried out in two stages: (i) estimation of the limiting threshold and (ii) application of the limiting function on the coefficients. The limiting thresholds and the limiting function should suppress the artifacts without distorting the ECG. For ECG records with sampling frequency of 360 Hz, it is observed that MA contributes to the magnitude of detailed coefficients in the scales 3 – 8, and hence the limiting is applied only to these scales.

For estimating the limiting thresholds, ECG record is divided into segments of two cardiac cycles and threshold for each segment is determined from the statistics of the coefficients and an MA control parameter η with the range of (0 – 1) as the following:

$$\varphi_j = \mu_j - \eta\sigma_j \tag{4.5}$$

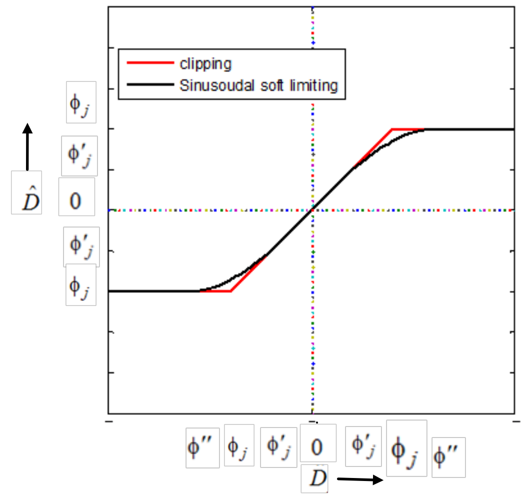

Figure 4.3: Smooth limiting function of Equation 4.6 and clipping.

where μ_j and σ_j are the mean and standard deviation of the magnitude of coefficients of the j th scale in the segment.

Limiting of the detail coefficients can be carried out by clipping the supra-threshold coefficients to the threshold values. For avoiding the possibility of introducing any significant distortion during the MA suppression, a sine-based smooth limiting function with two thresholds ϕ'_j and ϕ''_j and as given below is used:

$$\hat{D}_j(i) = \begin{cases} D_j(i), & |D_j(i)| < \phi'_j \\ \operatorname{sgn}(D_j(i))\left(\phi'_j + (\phi_j - \phi'_j)\sin\left(\dfrac{|D_j(i) - \phi'_j|\pi/2}{(\phi''_j - \phi'_j)}\right)\right), & \phi'_j \le |D_j(i)| \le \phi''_j \\ \operatorname{sgn}(D_j(i))\phi_j, & \phi''_j < |D_j(i)| \end{cases} \tag{4.6}$$

The function is shown in Figure 4.3. The thresholds ϕ'_j and ϕ''_j form the transition span as a quarter-cycle of a sine wave and are obtained as $\phi'_j = \phi_j - \sigma_j$ and $\phi''_j = \phi_j + \sigma_j$.

4.3 Test results

Wavelet-based denoising technique as described in the previous section was applied on a large number of noisy ECG records generated by adding noise-free ECG and ECG-free noise from the MIT-BIH database. EMG artifact was suppressed using the thresholding and motion artifact was suppressed using the limiting of the wavelet coefficient.

Figures 4.4 and 4.5 show the results of processing for an ECG segment with simulated noise. In Figure 4.4, ECG is corrupted by the EMG noise at the 0 dB. Duration of corresponding signals in database is from 19.4 s to 33.3 s Processed output shows effective noise suppression. It is observed that the amplitude of ECG and beat-to-beat relation in the shape of ECG signal remain unaffected. Morphological features can be more clearly observed. In Figure 4.5, ECG is corrupted by MA at −5 dB. Duration of corresponding signals in database is from 33.3s to 47.2 s The output waveform shows that MA is reduced considerably, Morphological features can be observed more clearly. However, amplitude of ECG gets attenuated. In both these processing examples, SNR_{impr} of approximately 9 dB was obtained.

Examples of the processing of ambulatory ECG are shown in Figures 4.6 and 4.7, for EMG+BW and MA, respectively, using empirically selected denoising control parameters. It is seen that the processed outputs in both the figures show effective noise suppression, beat-to-beat relations remain unaffected, and the morphological features are more clearly observed after denoising. In Figure 4.6, of no significant difference in R-peaks is observed after EMG denoising. Duration of corresponding signals is from 20 s to 43.1 s in database. In Figure 4.7, the MA is significantly, although not completely, suppressed with an attenuation of QRS complex affected by MA. Entire duration of ECG is displayed

Results of the detailed investigations on the effect of denoising control parameters using subjective and objective evaluation are presented in the next chapter.

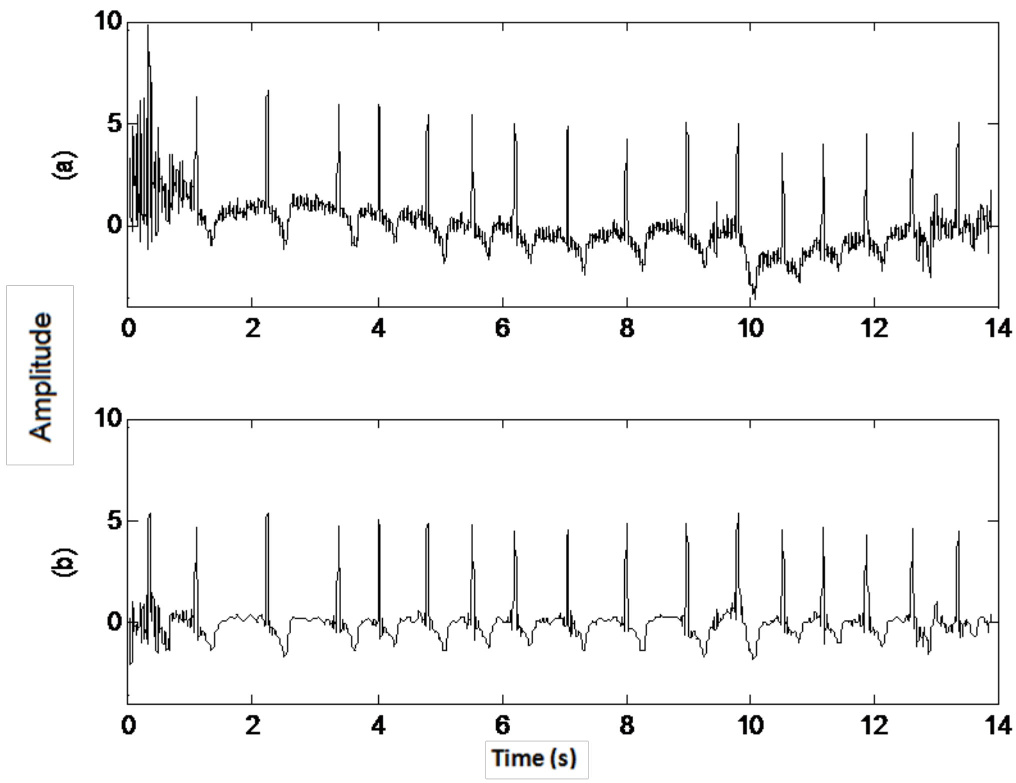

Figure 4.4: Suppression of EMG noise (a) ECG record-219 corrupted by EMG noise at input SNR = 0 dB and (b) processed output with $SNR_{impr.}$= 9.18 dB with $\varepsilon = 0.3$

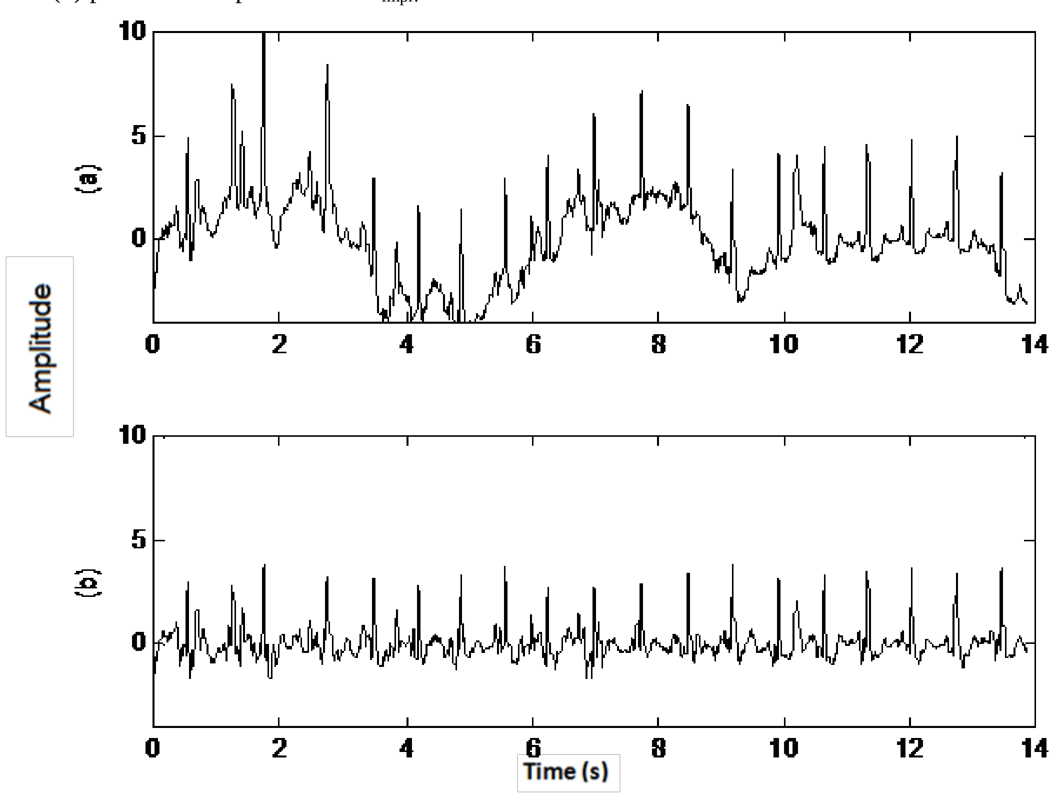

Figure 4.5: Suppression of MA (a) ECG record-105 corrupted by MA at input SNR =− 6 dB and (b) processed output ECG with $SNR_{impr.}$= 9.64 dB with $\eta = 0.5$

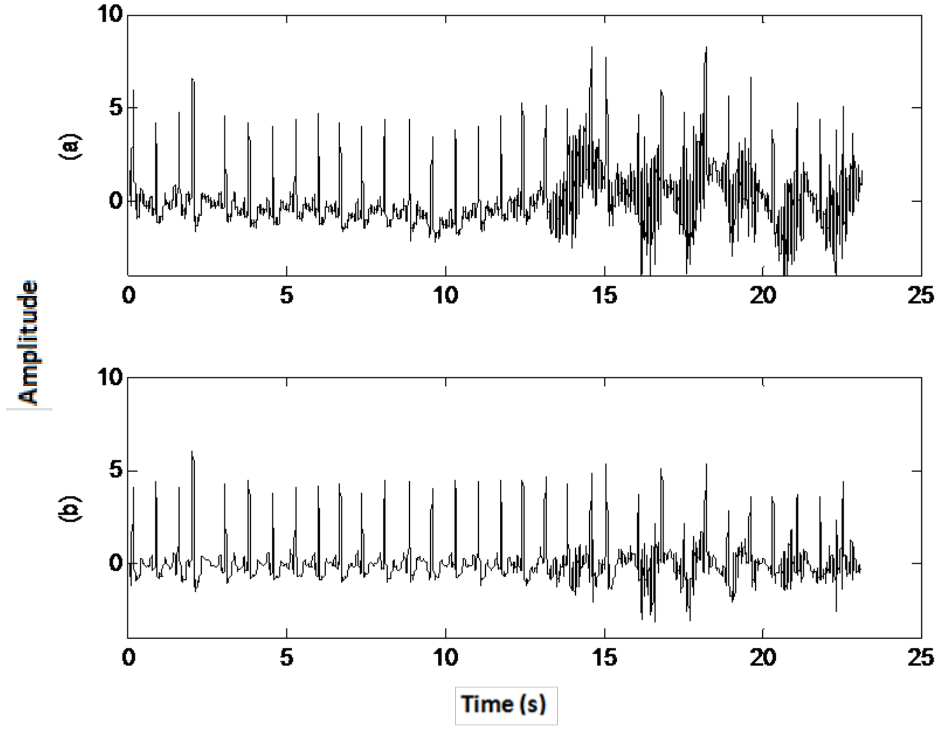

Figure 4.6: Suppression of EMG noise and BW: (a) ECG record-105 corrupted with EMG, (b) processed output with $SNR_{impr.}$= 6.1 dB using ε = 0.1

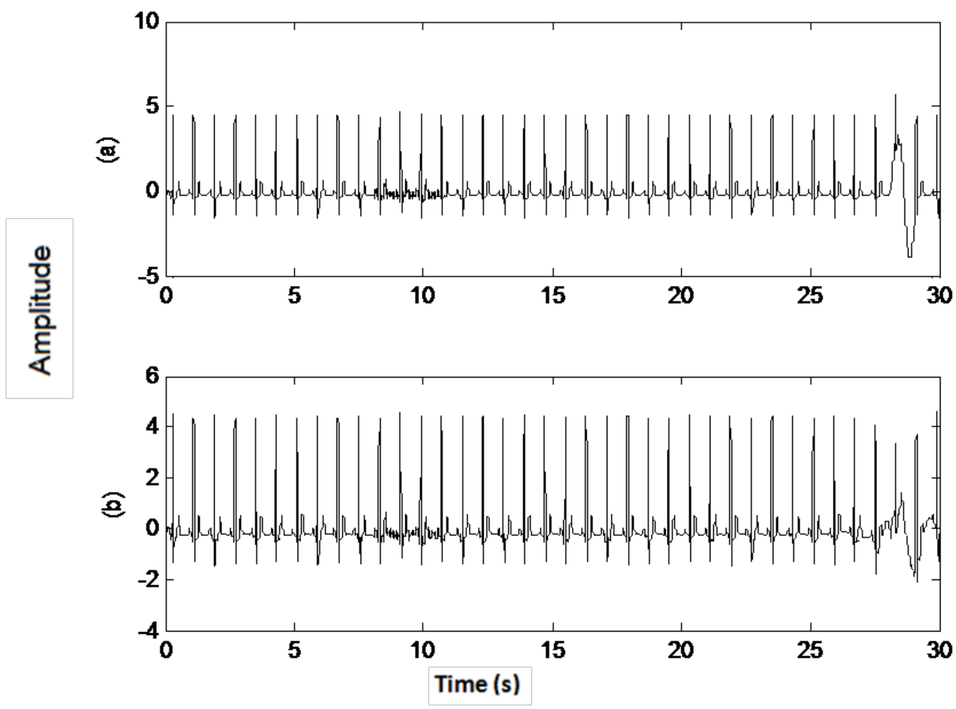

Figure 4.7: Suppression of MA: (a) entire ambulatory ECG with MA and (b) processed output. using η = 0.01.

Chapter 5

TESTS AND RESULTS

5.1 Introduction

Evaluation of the denoising technique, as described in the previous chapter, was carried out by applying it on ECG records with simulated artifacts as well as on ECG records with real artifacts. The objective evaluation has been carried out using the performance indices of SNR_{impr}, Corr. coeff., E-CDF, improvement indices based on L2-norm, max-min, skewness, and kurtosis, and output error decomposition coefficients of β (signal scaling coefficient), γ (noise attenuation coefficient), and κ (distortion coefficient) as described in the third chapter. Another performance measure involved errors in R-peak detection as a function of temporal tolerance. Programs for calculating these indices were developed in Matlab.

Section 5.2 describes the ECG records used in evaluating the denoising technique. Results based on the performance indices for objective evaluation of the denoising technique are presented in Section 5.3. Results based on errors in R-peak detection as a function of the temporal tolerance are presented in Section 5.4. Section 5.5 presents the results based on visual inspection of the denoised outputs for ambulatory ECG records with real artifacts. The results are discussed in the last section.

5.2 Generation of ECG records with simulated artifacts

Evaluation of the denoising technique was carried out by applying denoising technique on the ECG records with different simulated artifacts and different SNR values. These ECG record were generated by adding artifact-free ECG segments of one-minute duration and ECG-free artifact segments of the same duration. Both the ECG and artifact records were from MIT-BIH database as described in Section 3.2. The records were normalized to have RMS value of one and the records with simulated noise was generated at different SNRs using Equations 3.9 and 3.10 as described in Section 3.2. The one-minute segments from the 15 ECG records of 'mitdb' database were used: 100, 101, 103, 105, 106, 116, 118, 119, 123, 202, 203, 210, 213, 220, 232.

Four types of noises were used: (a) white Gaussian random signal, (b) BW 'bwm.mat', (c) EMG noise (muscle artifact 'mam.mat'), and (d) MA (electrode muscle 'emm.mat'). ECG records with simulated noise were generated for SNR values of 12, 6, 0, −6, and − 12 dB. These records were used for objective evaluation of the denoising technique for its application with different sets of control parameters.

The denoising was applied on ambulatory ECG recorded using a Holter monitor. These records were obtained during activities of daily life like limb movement, walking, climbing stairs, etc., and have related artifacts. The subjective evaluation was carried out by visual inspection of the processed outputs.

5.3 Results for ECG signals with simulated artifacts

Results of objective evaluation using performance indices for different combination of the denoising control parameters are presented in Tables 5.1, 5.2, 5.3, and 5.4 for ECG corrupted with white noise, BW, EMG, and MA, respectively. The following combinations of the EMG control parameter "emgCtrl" (ε) and the MA control parameter "maCtrl" (η) were used:

(a) ε = 0, η = 0;

(b) ε = 0.25, η = 0; (c) ε = 0.50, η = 0; (d) ε = 0.75, η = 0; (e) ε = 1, η = 0;

(f) ε = 0, η = 0.25; (g) ε = 0, η = 0.50; (h) ε = 0, η = 0.75; (i) ε = 0, η = 1;

(j) ε = 0.25, η = 0.25; (k) ε = 0.50, η = 0.50; (l) ε = 0.75, η = 0.75; (m) ε =1, η = 1.

The mean and standard deviation of the values of SNR_{impr},, Corr. coeff., β, γ, κ, II(RMS), II(MM), II(Skew.), II(kurt.), E(CDF)_out and E(CDF)_in are reported for each ε-η combination as a sub-table. These values are obtained over the 15 noisy ECG records.

The results in Table 5.1 are for ECG corrupted with white noise at various SNR values and with different denoising control parameters. In Table 5.1(a), ε = 0 and η = 0 corresponds to setting $D_1(n)$ and the $A_8(n)$ to zero and no modification of other coefficients. In terms of SNR_{impr}, there is a signal degradation of –2.1 dB at input SNR of 12 dB, the degradation decreases as input SNR decreases, and there is SNR_{impr} of 3 dB at input SNR of –12 dB. As reported in Table 3.3(a) in the third chapter, values of the correlation coefficient for noisy signal are 0.97, 0.89, 0.70, 045, and 0.24 for SNR of 12, 6, 0, – 6, – 12 dB, respectively. The corresponding values after denoising are 0.94, 0.90, 0.77, 0.54, and 0.31. Thus the result show that denoising results in signal degradation at high input SNR and modest improvement at low SNR. The values of post-denoising decomposition parameters (β, γ, κ) give decomposition of the errors after denoising. It is seen that β remains approximately 0.91 for all SNRs, indicating a signal attenuation. This may be attributed to loss of the signal components associated with $D_1(n)$ and $A_8(n)$ which were set to zero. The value of γ was found to be approximately 0.49 for all SNRs indicating an attenuation of noise due to $D_1(n)$ and $A_8(n)$ being set to zero. As input SNR decreases, the input noise increases and therefore even a fixed noise attenuation results in increase in SNR_{impr}. The values κ indicate that the denoising introduces significant distortion which increases with decrease in input SNR. Improvement indices being near to unity indicate that the denoising technique is effective in suppressing the artifacts. The values in table show that denoising was not effective in removing the white noise. Results in Table 5.1(b)-(e) give the values of the performance indices for $\eta = 0$, and ε increasing from 0.25 to 1. We see that with increase in ε, the signal degradation at high SNR and signal improvement at low SNR both progressively increase. With increase in ε, β decreases indicating increase in signal attenuation, γ decreases indicating noise attenuation, and κ increases indicating increase in distortion. The overall results show that EMG denoising technique results in decrease in white noise but its advantage is partly offset by signal attenuation and distortion. Results in Table 5.1(f)-(m) lead to similar conclusion about use of other combination of denoising control parameters.

The results in Table 5.2 are for denoising of ECG corrupted with BW at various SNRs and using different combinations of ε and η. In Table 5.2(a), $\varepsilon = 0$ and $\eta = 0$ corresponds to setting $D_1(n)$ and the $A_8(n)$ to zero and no modification of other coefficients. In terms of SNR_{impr}, there is a signal degradation of −0.2 dB at input SNR of 12 dB, the degradation decreases as input SNR decreases, and there is SNR_{impr} of 17.6 dB at input SNR of −12 dB. The correlation coefficient values also show a modest degradation at high SNRs and significant improvement at low SNRs. It is seen that β remains approximately 0.91 for all SNRs, indicating a signal attenuation. This may be attributed to loss of the signal components associated with $D_1(n)$ and $A_8(n)$ which were set to zero. The value of γ was found to be approximately 0.01 for all SNRs indicating a large attenuation of noise. The values κ indicate that the denoising introduces some distortion which gradually increases with decrease in input SNR. Improvement indices being near to unity indicate that the denoising technique is effective in suppressing the artifacts. Results in Table 5.2(b)-(e) give the values of the performance indices for $\eta = 0$, and ε increasing from 0.25 to 1. Values indicates that with increase in ε more signal has been distorted. We see that with increase in ε, the SNR_{impr} decrease at all the combination. With increase in ε, β decreases indicating increase in signal attenuation, γ decreases indicating noise attenuation, and κ gradually increases indicating increase in distortion. The overall results show that EMG denoising technique results in a significant suppression of BW. Results in Table 5.1(f)-(m) lead to similar conclusion about use of other combination of denoising control parameters and that maximum signal improvement is achieved for BW when $\varepsilon = 0$ and $\eta = 0$ at all the SNRs.

The results in Table 5.3 are for denoising of ECG corrupted with EMG noise at various SNRs and using different combinations of ε and η. With $\varepsilon = 0$ and $\eta = 0$ in Table 5.3(a), the values of SNR_{impr},show that there is a signal degradation of −1.6 dB at input SNR of 12 dB, the degradation decreases as input SNR decreases, and there is SNR_{impr} of 4.5 dB at input SNR of −12 dB. Thus the result show that denoising results in some signal degradation at high input SNR and some improvement at higher SNR. The correlation coefficient values also show a modest degradation at high SNRs and significant improvement at low SNRs. β, γ, κ give decomposition of the errors after denoising. It is seen that β remains approximately 0.91 for all SNRs, indicating a signal attenuation. The value of γ was found to be approximately 0.35 for all SNRs indicating an attenuation of noise due to $D_1(n)$ and $A_8(n)$ being set to zero. As input SNR decreases, the input noise increases and therefore even a fixed noise attenuation results in increase in SNR_{impr}. The values κ indicate that the denoising introduces some distortion which increases with decrease in input SNR. Values of improvement indices in the table show that denoising was the this combination is not effective in removing the EMG noise. Results in Table 5.3(b)-(e) give the values of the performance indices for $\eta = 0$, and ε increasing from 0.25 to 1. Values indicates that with increase in ε more signal has been distorted. With increase in ε, β decreases gradually indicating increase in signal attenuation, γ decreases indicating noise attenuation, and κ gradually increases indicating increase in distortion. The overall results show that EMG denoising technique results in decrease in EMG noise. Results in Table 5.1(f)-(m) lead to similar conclusion about use of other combination of denoising control parameters. The results show that use of high values of η (which is meant for MA suppression) increase EMG suppression in case of low input SNR, but result in signal degradation in case of high input SNR. Therefore the values of ε should be selected on the basis of estimation of the level of EMG corruption and η should be kept low unless significant MA is estimated to be present.

The results in Table 5.4 are for denoising of ECG corrupted with MA at various SNRs and using different combinations of ε and η. With $\varepsilon = 0$ and $\eta = 0$ in Table 5.4(a), the values of SNR_{impr},show that there is a signal degradation of −1.2 dB at input SNR of 12 dB, the degradation decreases as input SNR decreases, and there is SNR_{impr} of 5.9 dB at input SNR of −12 dB. Thus the result show that denoising results in some signal degradation at high input SNR and some improvement at higher SNR. The correlation coefficient values also show a modest degradation at high SNRs and significant improvement at low SNRs. β, γ, κ give decomposition of the errors after denoising. It is seen that β remains approximately 0.91 for all SNRs, indicating a signal attenuation. The value of γ was found to be approximately 0.24 for all SNRs indicating an attenuation of noise due to $D_1(n)$ and $A_8(n)$ being set to zero. As input SNR decreases, the input noise increases and therefore even a fixed noise attenuation results in increase in SNR_{impr}. The values κ indicate that the denoising introduces some distortion which increases with decrease in input SNR. Values of improvement indices in the table show that denoising was the this combination is not effective in removing the EMG noise. Results in Table 5.3(b)-(e) give the values of the performance indices for $\eta = 0$, and ε increasing from 0.25 to 1. The values in the table show that with increase in ε, γ decreases indicating noise attenuation but decrease in β indicates significant signal attenuation and increase in κ indicates increase in distortion. As a result, SNR_{impr} increases only slightly with increase in ε. Results in Table 5.3(f)-(i) give the values of the performance indices for $\varepsilon = 0$, and η increasing from 0.25 to 1. The results indicate the signal degrades at higher values of input SNR and signal is enhanced for lower values of the input SNR. Results in Table 5.1(j)-(m) lead to similar conclusion about use of other combination of denoising control parameters. The results show that use of high values of ε are not very useful for MA suppression, while high values of η significantly suppress the MA.

We may consider a combination of control parameters which gives large SNR_{impr} at low input SNR without a very large signal degradation at high input SNR as an optimum combination. For BW-corrupted ECG at SNR = −12 dB, the SNR_{impr} after denoising using $\varepsilon = 0$ and $\eta = 0$ is 17.6 dB. Some of the other combinations result in a slightly higher improvement but at the expense of signal degradation for signals with high input SNR. Therefore, the combination $\varepsilon = 0$ and $\eta = 0$ may be considered as the optimum combination for BW suppression. Although it results in large SNR_{impr} there is a scope for further improvement if the signal attenuation and distortion associated with the denoising process can be reduced. This possibly can be done by applying level-dependent thresholding on $D_1(n)$ and $A_8(n)$, rather than setting them to zero. For EMG-corrupted ECG, $\varepsilon = 0.5$ and $\eta = 0$ may be considered as optimal, with SNR_{impr} of 5.9 dB and –4.6 dB at input SNR of –12 and 12 dB, respectively. For MA-corrupted ECG, $\varepsilon = 0$ and $\eta = 0.5$ may be considered as optimal, with SNR_{impr} of 9.3 dB and –3.3 dB at input SNR of –12 and 12 dB, respectively. Depending on the input SNR other combinations may result in higher SNR_{impr}. Therefore, selection of the combination of denoising control parameters should be made based on an assessment of the level and type of artifacts: a higher ε for high level of EMG noise and higher η for high level of MA.

5.4 Errors in R-peak detection

Evaluation of the denoising technique was carried out with the detection and insertion errors in R-peak detection as a function of temporal tolerance. This evaluation is carried out for a one-minute segment of ECG-105 and corrupted with BW, EMG, and MA. The errors were calculated, with reference to R-peaks detected in the noise-free ECG, for noisy ECG and ECG denoised using the optimal set of control parameters as established on the basis of SNR_{impr} at the end of previous section. These parameters combinations are as the following:

BW: $\varepsilon = 0$, $\eta = 0$;

EMG noise: $\varepsilon = 0.5$, $\eta = 0$;

MA: $\varepsilon = 0$, $\eta = 0.5$

The results are given in Tables 5.5, 5.6, and 5.7 for BW, EMG noise, and MA, respectively.

Results for BW-corrupted ECG in Table 5.5(a) confirm that the Pan-Tompkins algorithm for R-peak detection has a very low sensitivity to the BW. For input SNR of 0 dB and higher, there are no errors for tolerance of 10 ms and larger. For lower SNR there is a single insertion error and no detection errors. Results after denoising given in Table 5.5(b) do not show any changes in the detection and insertion errors.

Results for EMG-corrupted ECG in Table 5.6(a) show large insertion errors for input SNR of 0 dB and below and large detection errors for input SNR of –6 dB and below. Denoising is unable to decrease the errors. The result show that even a mild level of EMG noise disrupts R-peak detection by Pan-Tompkins algorithm and the denoising technique does not suppress the noise sufficiently.

Results for MA-corrupted ECG in Table 5.7(a) show large insertion errors for input SNR of –6 and – 12 dB and large detection errors for input SNR of –12 dB. Thus, as compared to EMG noise, MA poses less of a challenge in R-peak detection. Denoising results in slight increase in insertion errors and significant decrease in detection errors.

5.5 Results for ambulatory ECG with real artifacts

There are many techniques which are efficient in artifact suppression for ECG signals in normal subjects which may fail in presence of arrhythmias or other heart conditions. Hence it becomes very important to inspect the performance of the denoising technique in arrhythmias or other abnormal heart conditions. Effective artifact suppression has been observed by visual inspection for ambulatory ECG signals with arrhythmia, ST deviation, ventricular tachycardia, atrial flutter.

Figure 5.1 shows suppression of artifact in ECG recorded in walking condition by Holter monitoring. From figure it can be seen that amplitude of ECG signal remained the same. Beat-to-beat relation in the shape of ECG was observed. Morphological features can be seen. Presence of artifacts is reduced considerably. In Figure 5.2 shows suppression of artifact in ECG recorded in sitting condition by Holter monitoring. Processed waveform shows good suppression of the artifact except amplitude of ECG is somewhat attenuated retaining the original features of ECG. Morphological features could be observed. Presence of artifacts is reduced considerably. Figure 5.3 shows suppression artifact in patient's ECG with atrial fibrillation using both thresholding and clipping. Amplitude of ECG signal remained the almost same. Beat-to-beat relation in the shape of ECG was observed. Although the shape of ECG has changed. In the same noisy ECG that is used in Figure 5.5 use only thresholding the resulted in a Figure 5.4. Processed waveform shows good suppression of the artifact amplitude of ECG is even less attenuated. More importantly the original features of ECG are retained. This shows that motion artifact may fail in some conditions. Figure 5.5 is a noisy ECG signal from sudden death database. The input unprocessed ECG is corrupted by heavy noise and denoising had reduced the noise significantly. ECG is not attenuated by the denoising technique and the ECG features can be observed more clearly. Beat-to-beat change observed is somewhat more as compared to other figures.

Figure 5.8 shows suppression of artifact in patient's ECG with malignant ventricular ectopy. In Figure 5.9 shows suppression of artifact in patient's ECG from ventricular tachycardia. In Figure 5.10 shows suppression of artifact in patient's ECG with atrial fibrillation. Figure 5.11 shows suppression of artifact in patient's ECG with ST-change. Processed ECG seems to be free from all the artifact. There is slight attenuation in ECG. Morphological features can be seen.

Beat-to-beat repeatability is observed. As can be observed in Figures 5.3-5.11 processed ECG is always more smoother than input ECG. Significant attenuation in ECG amplitude is not observed in any of these figures. Beat-to-beat repeatability is observed in most ECGs. As can be observed in Figures 5.3-5.11 processed ECG is always smoother than the input ECG. Noise is seen to be suppressed by the denoising technique.

5.6 Summary

Subjective and objective evaluation methods have been used to evaluate the performance of the denoising control parameter as the described in the ECG technique. Adding signals and artifact at varying levels of SNR from MIT-BIH database following objective parameter were reported for the different signals and artifacts: SNR_{impr}, Corr. Coef., β, γ, κ, II(RMS), II(MM), II(skewness), II(kurtosis), E(CDF)_in, and E(CDF)_out. Along with that visual inspection was carried out on the several types of the signal including normal ECG, ECG with arrhythmia, ST deviation, ventricular tachycardia, atrial flutter. Temporal tolerance for a ECG signal with different artifacts along with different parameters was reported.

It was observed that the SNR_{impr} increases for all the signal with decrease in SNR. Correlation coefficient has improved for the lower SNRs as compared to the denoised signal while decreased for very higher SNRs as parameters were set arbitrarily and not to optimize it. β values seems to be almost constant across the entire values of SNR and all types of artifact. Beta values are lower as compared to unprocessed output. Value of β is almost constant from higher SNR to lower SNR. Value of γ for baseline wander artifact is found approximately i.e. 0.01 across all the artifacts. Value of κ increases with decrease in SNR indicating increased distortions for denoising. II(RMS) and II(MM) came more closer to one with decrease in SNR. E(CDF)_out was less than E(CDF)_in at lower SNR indicating effective denoising technique. At very high SNR it is vice-versa indicating distortion to be introduced or parameters were not optimized for it. Test with temporal accuracy showed that denoising technique with lower parameters do not introduced distortion comparable to R-peak and neither attenuated the R-peak sufficiently to be missed. It also indicated that different SNR with different artifact has different optimum parameters. This indicates that motion artifact may distort a signal.

It is seen that different combinations of "emgCtrl" and "maCtrl" are found to be optimum. Low non-zero values of "emgCtrl" are well suited. In such cases high value of "emgCtrl" results in distortion, degrading the quality of the output processed signal. At very low SNRs higher values of "emgCtrl" give better SNR_{impr}. In such cases if "emgCtrl" is low adequate artifact suppression do not take place. At higher values of input SNR "maCtrl" should be zero. In such cases high value "maCtrl" results in distortion. At very low SNRs high values of "maCtrl" gives the better SNR_{impr}. In such cases low "emgCtrl" value does not result in adequate artifact suppression. If both the parameters are high than signal denoising is optimum at lower values of SNR.

As techniques which are efficient in artifact suppression for ECG signals in normal subjects fail in presence of arrhythmias or other heart conditions, the performance of the denoising technique was tested for ECG from patients with arrhythmias or other abnormal heart conditions. Effective artifact suppression has been observed by visual inspection for normal ECG, ECG ambulatory ECG signals with arrhythmia, ST deviation, ventricular tachycardia, malignant ventricular ectopy, and atrial flutter except that in the case of atrial flutter the denoising with a non-zero "maCtrl" significantly changed the ECG shape.

Table 5.1: Results for ECG with white noise

a) ε= 0,η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-2.060	2.339	0.974	1.332	2.382	0.525	2.870	0.198	3.010	0.065
Corr.Coeff.	0.939	0.040	0.896	0.041	0.768	0.044	0.539	0.038	0.308	0.027
β	0.913	0.073	0.914	0.072	0.914	0.076	0.916	0.073	0.912	0.082
γ	0.495	0.027	0.500	0.013	0.497	0.008	0.496	0.008	0.494	0.004
κ	0.288	0.084	0.363	0.068	0.567	0.048	1.032	0.027	2.007	0.016
II(RMS)	1.942	1.287	0.839	0.295	0.548	0.078	0.432	0.014	0.369	0.005
II(MM)	1.049	1.259	0.659	0.309	0.436	0.133	0.365	0.090	0.336	0.085
II(Skew.)	1.082	1.142	0.554	0.310	0.302	0.094	0.102	0.040	0.022	0.011
II(Kurt.)	1.915	1.299	0.774	0.340	0.321	0.053	0.084	0.012	0.011	0.008
E(CDF)_out	0.016	0.038	0.024	0.041	0.051	0.050	0.121	0.126	0.131	0.166
E(CDF)_in	0.011	0.016	0.053	0.066	0.100	0.091	0.181	0.151	0.189	0.169

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b) ε= 0.25 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-3.678	1.476	1.051	1.208	3.668	0.682	4.412	0.277	4.493	0.102
Corr.Coeff.	0.919	0.035	0.892	0.037	0.795	0.041	0.579	0.038	0.334	0.025
β	0.826	0.055	0.832	0.055	0.840	0.059	0.849	0.064	0.841	0.063
γ	0.307	0.037	0.327	0.039	0.353	0.014	0.380	0.010	0.393	0.004
κ	0.337	0.053	0.381	0.050	0.528	0.041	0.918	0.030	1.780	0.020
II(RMS)	4.265	0.935	1.577	0.240	0.867	0.057	0.624	0.019	0.513	0.004
II(MM)	1.536	0.875	0.890	0.264	0.573	0.206	0.469	0.086	0.421	0.081
II(Skew.)	0.337	1.535	0.539	0.420	0.422	0.140	0.161	0.051	0.037	0.017
II(Kurt.)	1.484	1.868	0.838	0.476	0.484	0.108	0.180	0.040	0.068	0.038
E(CDF)_out	0.021	0.040	0.020	0.031	0.029	0.028	0.088	0.100	0.110	0.152
E(CDF)_in	0.011	0.017	0.053	0.067	0.101	0.092	0.180	0.151	0.189	0.168

c) ε= 0.5 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-4.447	1.143	0.562	0.939	3.645	0.619	4.698	0.280	4.909	0.106
Corr.Coeff.	0.905	0.032	0.880	0.032	0.788	0.036	0.578	0.036	0.337	0.025
β	0.786	0.047	0.794	0.046	0.808	0.049	0.813	0.058	0.804	0.064
γ	0.287	0.041	0.295	0.028	0.329	0.019	0.348	0.011	0.358	0.006
κ	0.357	0.044	0.396	0.040	0.535	0.034	0.911	0.026	1.739	0.017
II(RMS)	5.124	0.956	1.833	0.213	0.940	0.057	0.671	0.019	0.552	0.007
II(MM)	1.596	0.827	0.902	0.359	0.688	0.167	0.483	0.095	0.413	0.072
II(Skew.)	-0.058	2.269	0.471	0.630	0.405	0.214	0.167	0.062	0.028	0.031

II(Kurt.)	1.016	1.953	0.720	0.499	0.461	0.131	0.180	0.045	0.079	0.044
E(CDF)_out	0.023	0.040	0.024	0.032	0.029	0.025	0.083	0.097	0.103	0.147
E(CDF)_in	0.011	0.017	0.053	0.066	0.101	0.093	0.178	0.148	0.190	0.170

d) ε= 0.75 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-5.358	0.918	-0.101	0.776	3.659	0.506	5.233	0.228	5.703	0.081
Corr.Coeff.	0.883	0.029	0.860	0.030	0.777	0.031	0.578	0.031	0.343	0.021
β	0.740	0.049	0.746	0.048	0.754	0.046	0.758	0.050	0.749	0.052
γ	0.225	0.029	0.255	0.025	0.273	0.015	0.295	0.009	0.301	0.006
κ	0.384	0.034	0.420	0.031	0.544	0.026	0.890	0.018	1.664	0.009
II(RMS)	6.315	1.230	2.119	0.272	1.073	0.065	0.749	0.021	0.619	0.009
II(MM)	2.135	1.810	1.049	0.366	0.687	0.151	0.517	0.107	0.455	0.050
II(Skew.)	-0.841	2.369	0.254	0.687	0.375	0.201	0.178	0.066	0.037	0.019
II(Kurt.)	0.477	2.230	0.561	0.520	0.455	0.158	0.206	0.056	0.121	0.074
E(CDF)_out	0.027	0.043	0.025	0.030	0.026	0.020	0.071	0.089	0.094	0.140
E(CDF)_in	0.011	0.016	0.053	0.067	0.099	0.091	0.181	0.152	0.189	0.168

e) ε= 1 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-6.335	0.755	-0.758	0.651	3.540	0.459	5.799	0.239	6.531	0.131
Corr.Coeff.	0.853	0.028	0.835	0.027	0.761	0.030	0.587	0.031	0.346	0.025
β	0.685	0.053	0.694	0.051	0.701	0.051	0.715	0.050	0.684	0.056
γ	0.188	0.032	0.212	0.020	0.228	0.012	0.242	0.008	0.246	0.007
κ	0.414	0.026	0.441	0.023	0.548	0.020	0.855	0.016	1.570	0.016
II(RMS)	7.452	1.431	2.432	0.327	1.196	0.083	0.825	0.021	0.687	0.011
II(MM)	2.521	1.563	1.236	0.570	0.758	0.136	0.565	0.116	0.471	0.082
II(Skew.)	-1.612	2.919	0.161	1.086	0.369	0.328	0.200	0.115	0.036	0.031
II(Kurt.)	-0.325	2.261	0.427	0.615	0.450	0.210	0.254	0.079	0.159	0.094
E(CDF)_out	0.031	0.042	0.028	0.028	0.025	0.016	0.060	0.076	0.083	0.130
E(CDF)_in	0.011	0.017	0.053	0.067	0.100	0.090	0.179	0.150	0.190	0.169

f) ε= 0 η= 0.25

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-3.170	2.353	0.411	1.479	2.227	0.634	2.897	0.152	3.096	0.067
Corr.Coeff.	0.922	0.049	0.871	0.055	0.729	0.063	0.496	0.050	0.280	0.032
β	0.802	0.107	0.803	0.107	0.805	0.108	0.809	0.100	0.814	0.099
γ	0.485	0.025	0.486	0.010	0.489	0.008	0.487	0.006	0.488	0.004

κ	0.288	0.064	0.361	0.053	0.560	0.038	1.021	0.018	1.990	0.014
II(RMS)	5.326	2.390	1.691	0.589	0.758	0.143	0.488	0.033	0.388	0.006
II(MM)	10.802	22.316	1.393	0.816	0.682	0.277	0.488	0.166	0.374	0.078
II(Skew.)	1.323	1.659	0.536	0.445	0.226	0.145	0.076	0.033	0.013	0.017
II(Kurt.)	1.938	1.589	0.679	0.492	0.245	0.111	0.051	0.018	0.009	0.008
E(CDF)_out	0.026	0.043	0.028	0.040	0.046	0.044	0.114	0.120	0.129	0.166
E(CDF)_in	0.012	0.017	0.054	0.068	0.099	0.091	0.180	0.151	0.188	0.169

g) ε= 0 η= 0.5

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-4.005	2.225	-0.044	1.469	2.107	0.663	2.901	0.205	3.116	0.061
Corr.Coeff.	0.911	0.052	0.854	0.060	0.705	0.072	0.471	0.059	0.264	0.037
β	0.740	0.119	0.742	0.117	0.751	0.119	0.756	0.113	0.760	0.113
γ	0.487	0.023	0.485	0.011	0.486	0.008	0.486	0.008	0.487	0.003
κ	0.287	0.052	0.358	0.043	0.556	0.030	1.016	0.018	1.980	0.012
II(RMS)	7.099	2.929	2.149	0.706	0.861	0.162	0.516	0.035	0.396	0.008
II(MM)	5.932	7.117	1.513	0.763	0.804	0.312	0.498	0.151	0.394	0.064
II(Skew.)	1.239	2.526	0.363	0.710	0.166	0.178	0.049	0.043	0.010	0.016
II(Kurt.)	2.345	2.355	0.631	0.485	0.202	0.133	0.038	0.022	0.008	0.006
E(CDF)_out	0.032	0.049	0.031	0.039	0.043	0.040	0.112	0.119	0.129	0.167
E(CDF)_in	0.012	0.017	0.053	0.067	0.099	0.090	0.180	0.151	0.189	0.168

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h) ε= 0 η= 0.75

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-5.515	2.112	-0.990	1.518	1.745	0.768	2.850	0.215	3.203	0.069
Corr.Coeff.	0.876	0.069	0.813	0.080	0.656	0.092	0.432	0.071	0.244	0.042
β	0.642	0.139	0.650	0.137	0.662	0.138	0.674	0.130	0.690	0.128
γ	0.474	0.019	0.476	0.014	0.479	0.005	0.479	0.008	0.478	0.006
κ	0.302	0.035	0.370	0.030	0.564	0.022	1.015	0.011	1.969	0.012
II(RMS)	9.817	3.441	2.795	0.889	1.003	0.188	0.550	0.044	0.413	0.012
II(MM)	4.135	2.957	1.868	1.018	0.954	0.371	0.536	0.134	0.453	0.050
II(Skew.)	-0.224	5.016	0.113	0.928	0.038	0.255	0.022	0.043	0.003	0.011
II(Kurt.)	2.214	2.341	0.548	0.657	0.121	0.155	0.023	0.020	0.005	0.006
E(CDF)_out	0.041	0.061	0.040	0.049	0.042	0.037	0.109	0.118	0.128	0.166
E(CDF)_in	0.012	0.017	0.053	0.067	0.100	0.091	0.179	0.151	0.191	0.168

i) ε= 0 η= 1

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-7.382	2.229	-2.422	1.833	1.120	0.965	2.675	0.342	3.210	0.096
Corr.Coeff.	0.780	0.167	0.711	0.170	0.575	0.140	0.366	0.100	0.209	0.045
β	0.528	0.173	0.528	0.172	0.551	0.168	0.556	0.168	0.584	0.132
γ	0.467	0.021	0.471	0.011	0.470	0.011	0.471	0.008	0.471	0.007
κ	0.353	0.067	0.411	0.059	0.583	0.030	1.025	0.022	1.969	0.017
II(RMS)	12.257	4.093	3.311	0.805	1.142	0.199	0.591	0.046	0.427	0.012
II(MM)	7.352	7.822	2.200	1.136	1.085	0.516	0.645	0.162	0.429	0.068
II(Skew.)	-2.604	6.244	-0.712	1.632	-0.140	0.343	-0.023	0.060	-0.002	0.026
II(Kurt.)	1.193	4.358	0.136	0.978	0.026	0.180	0.003	0.028	0.003	0.006
E(CDF)_out	0.049	0.071	0.060	0.066	0.044	0.034	0.104	0.114	0.125	0.167
E(CDF)_in	0.011	0.016	0.054	0.068	0.099	0.091	0.180	0.151	0.188	0.168

j) ε= 0.25 η= 0.25

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-4.532	1.583	0.485	1.365	3.475	0.832	4.464	0.317	4.647	0.116
Corr.Coeff.	0.908	0.042	0.874	0.048	0.759	0.058	0.535	0.055	0.300	0.030
β	0.711	0.082	0.720	0.084	0.727	0.087	0.741	0.090	0.729	0.075
γ	0.289	0.043	0.320	0.032	0.346	0.019	0.372	0.007	0.383	0.005
κ	0.308	0.040	0.353	0.038	0.505	0.031	0.895	0.023	1.743	0.018
II(RMS)	8.040	1.821	2.487	0.457	1.112	0.110	0.690	0.022	0.540	0.005
II(MM)	3.371	1.705	1.675	0.990	0.866	0.400	0.562	0.105	0.456	0.068
II(Skew.)	0.819	1.556	0.602	0.334	0.370	0.117	0.122	0.041	0.025	0.022
II(Kurt.)	1.622	2.124	0.813	0.510	0.405	0.159	0.134	0.038	0.064	0.040
E(CDF)_out	0.033	0.048	0.031	0.036	0.021	0.018	0.081	0.095	0.107	0.152
E(CDF)_in	0.011	0.017	0.053	0.066	0.099	0.091	0.180	0.151	0.188	0.169

k) ε= 0.5 η= 0.5

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-5.852	1.212	-0.479	1.142	3.230	0.798	4.727	0.314	5.140	0.106
Corr.Coeff.	0.891	0.040	0.852	0.048	0.728	0.063	0.503	0.056	0.284	0.034
β	0.607	0.077	0.617	0.084	0.630	0.088	0.644	0.084	0.643	0.081
γ	0.266	0.037	0.284	0.030	0.316	0.020	0.338	0.007	0.346	0.005
κ	0.293	0.030	0.340	0.026	0.489	0.020	0.870	0.015	1.675	0.014
II(RMS)	11.194	2.188	3.378	0.587	1.335	0.124	0.775	0.024	0.592	0.008
II(MM)	5.909	5.886	2.205	1.124	1.038	0.361	0.603	0.145	0.478	0.075
II(Skew.)	0.235	1.555	0.371	0.431	0.256	0.118	0.095	0.032	0.012	0.017
II(Kurt.)	1.362	1.967	0.686	0.592	0.323	0.153	0.126	0.052	0.078	0.055

E(CDF)_out	0.044	0.059	0.038	0.045	0.021	0.014	0.070	0.086	0.099	0.146
E(CDF)_in	0.011	0.016	0.052	0.065	0.100	0.090	0.179	0.151	0.188	0.167

l) ε= 0.75 η= 0.75

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-7.598	1.019	-2.006	0.893	2.572	0.703	5.232	0.279	6.137	0.131
Corr.Coeff.	0.846	0.044	0.798	0.053	0.662	0.073	0.457	0.060	0.256	0.035
β	0.470	0.089	0.472	0.084	0.475	0.089	0.505	0.085	0.504	0.076
γ	0.218	0.027	0.230	0.020	0.252	0.016	0.271	0.007	0.277	0.007
κ	0.283	0.029	0.327	0.021	0.461	0.020	0.809	0.012	1.545	0.018
II(RMS)	15.070	3.033	4.445	0.584	1.697	0.156	0.920	0.037	0.689	0.012
II(MM)	7.070	5.237	2.408	0.945	1.309	0.410	0.785	0.149	0.504	0.061
II(Skew.)	-1.404	2.643	-0.159	0.701	0.063	0.170	0.046	0.043	0.015	0.017
II(Kurt.)	0.774	2.159	0.498	0.827	0.237	0.168	0.147	0.075	0.134	0.096
E(CDF)_out	0.061	0.076	0.063	0.057	0.026	0.021	0.049	0.065	0.085	0.134
E(CDF)_in	0.011	0.017	0.052	0.066	0.100	0.090	0.180	0.151	0.189	0.169

m) ε= 1 η= 1

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-9.547	1.117	-3.775	1.127	1.366	0.903	5.268	0.459	7.030	0.210
Corr.Coeff.	0.691	0.160	0.634	0.168	0.504	0.148	0.353	0.095	0.202	0.039
β	0.304	0.109	0.308	0.114	0.299	0.110	0.327	0.102	0.341	0.069
γ	0.162	0.017	0.183	0.019	0.196	0.011	0.208	0.010	0.217	0.007
κ	0.297	0.059	0.339	0.055	0.449	0.038	0.742	0.020	1.404	0.031
II(RMS)	19.739	3.511	5.420	0.746	2.011	0.157	1.069	0.040	0.780	0.014
II(MM)	4.337	18.638	3.650	1.890	1.623	0.532	0.877	0.120	0.571	0.054
II(Skew.)	-5.528	4.477	-1.353	1.034	-0.252	0.301	-0.023	0.052	-0.005	0.053
II(Kurt.)	-1.155	3.655	-0.237	0.892	0.125	0.257	0.201	0.130	0.236	0.173
E(CDF)_out	0.077	0.086	0.082	0.080	0.040	0.029	0.028	0.040	0.070	0.117
E(CDF)_in	0.011	0.017	0.053	0.067	0.099	0.090	0.179	0.152	0.189	0.166

Table 5.2 Results for ECG with BW

a) ε= 0 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-0.164	3.286	5.645	3.176	10.979	2.820	15.138	2.013	17.571	1.000
Corr.Coeff.	0.955	0.039	0.953	0.039	0.949	0.039	0.930	0.040	0.865	0.042
β	0.913	0.073	0.913	0.074	0.913	0.074	0.912	0.074	0.911	0.076
γ	0.010	0.009	0.011	0.004	0.012	0.002	0.012	0.001	0.012	0.001
κ	0.258	0.092	0.263	0.091	0.281	0.085	0.343	0.072	0.516	0.049
II(RMS)	2.430	1.244	1.355	0.306	1.093	0.092	1.017	0.032	0.984	0.012
II(MM)	1.359	4.268	1.269	0.654	1.090	0.261	0.974	0.104	0.897	0.073
II(Skew.)	1.903	1.250	1.226	0.339	1.075	0.142	0.967	0.146	1.272	1.583
II(Kurt.)	2.562	1.367	1.482	0.440	1.208	0.218	1.077	0.153	0.860	0.132
E(CDF)_out	0.029	0.052	0.018	0.024	0.019	0.026	0.009	0.027	0.003	0.008
E(CDF)_in	0.016	0.021	0.051	0.059	0.133	0.111	0.264	0.184	0.123	0.223

b) ε= 0.25 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-3.332	1.535	2.603	1.519	8.337	1.431	13.457	1.205	17.067	0.756
Corr.Coeff.	0.925	0.034	0.924	0.034	0.919	0.034	0.901	0.034	0.838	0.036
β	0.818	0.053	0.817	0.053	0.817	0.053	0.817	0.054	0.819	0.055
γ	0.008	0.009	0.009	0.005	0.010	0.002	0.011	0.001	0.011	0.001
κ	0.326	0.053	0.329	0.053	0.341	0.051	0.385	0.045	0.527	0.035
II(RMS)	4.854	1.679	1.962	0.247	1.268	0.066	1.076	0.023	1.007	0.009
II(MM)	2.948	3.003	1.620	0.675	1.228	0.261	1.039	0.096	0.924	0.067
II(Skew.)	0.873	1.442	0.964	0.537	1.003	0.376	1.125	0.885	1.008	0.809
II(Kurt.)	1.474	1.562	1.153	0.534	1.051	0.264	0.970	0.189	0.803	0.160
E(CDF)_out	0.029	0.043	0.016	0.023	0.017	0.019	0.014	0.028	0.004	0.010
E(CDF)_in	0.016	0.021	0.051	0.059	0.133	0.111	0.264	0.184	0.123	0.223

(c) ε= 0.5 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-4.240	1.124	1.667	1.090	7.356	1.008	12.545	0.838	16.525	0.564
Corr.Coeff.	0.911	0.030	0.909	0.029	0.902	0.029	0.880	0.028	0.813	0.030
β	0.781	0.046	0.779	0.045	0.773	0.045	0.767	0.045	0.767	0.048
γ	0.008	0.009	0.009	0.004	0.010	0.002	0.010	0.001	0.011	0.001
κ	0.349	0.041	0.352	0.040	0.366	0.038	0.410	0.033	0.545	0.026
II(RMS)	5.743	1.997	2.192	0.253	1.342	0.065	1.104	0.023	1.019	0.009
II(MM)	3.400	2.742	1.690	0.738	1.247	0.276	1.050	0.122	0.936	0.070

II(Skew.)	0.423	1.932	0.849	0.769	0.977	0.598	1.237	1.562	0.771	0.128
II(Kurt.)	0.976	1.743	0.999	0.587	0.972	0.292	0.905	0.205	0.752	0.173
E(CDF)_out	0.033	0.043	0.019	0.026	0.020	0.020	0.017	0.028	0.004	0.012
E(CDF)_in	0.016	0.021	0.051	0.059	0.133	0.111	0.264	0.184	0.123	0.223

d) ε= 0.75 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-5.361	0.832	0.512	0.803	6.204	0.826	11.513	0.707	15.835	0.493
Corr.Coeff.	0.885	0.025	0.881	0.025	0.871	0.026	0.846	0.027	0.775	0.031
β	0.731	0.045	0.726	0.045	0.717	0.049	0.707	0.050	0.702	0.055
γ	0.008	0.009	0.009	0.004	0.010	0.002	0.010	0.001	0.010	0.001
κ	0.381	0.030	0.386	0.028	0.401	0.029	0.442	0.024	0.568	0.018
II(RMS)	6.815	2.571	2.465	0.330	1.424	0.084	1.133	0.029	1.031	0.012
II(MM)	3.535	2.912	1.737	0.793	1.279	0.296	1.069	0.134	0.958	0.071
II(Skew.)	-0.387	2.464	0.627	0.937	0.898	0.730	1.286	2.051	0.678	0.100
II(Kurt.)	0.277	2.112	0.790	0.691	0.875	0.333	0.830	0.234	0.696	0.187
E(CDF)_out	0.039	0.043	0.023	0.029	0.023	0.024	0.021	0.029	0.005	0.015
E(CDF)_in	0.016	0.021	0.051	0.059	0.133	0.111	0.264	0.184	0.123	0.223

e) ε= 1 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-6.165	0.787	-0.289	0.734	5.384	0.714	10.776	0.709	15.329	0.512
Corr.Coeff.	0.859	0.027	0.855	0.026	0.842	0.027	0.814	0.032	0.741	0.038
β	0.687	0.052	0.682	0.051	0.671	0.054	0.658	0.063	0.651	0.067
γ	0.008	0.009	0.009	0.004	0.010	0.002	0.010	0.001	0.010	0.001
κ	0.405	0.025	0.410	0.023	0.426	0.021	0.464	0.018	0.583	0.014
II(RMS)	7.711	3.102	2.690	0.398	1.488	0.100	1.156	0.038	1.040	0.015
II(MM)	3.962	3.089	1.838	0.833	1.304	0.308	1.085	0.129	0.975	0.068
II(Skew.)	-1.127	3.083	0.436	1.118	0.834	0.880	1.299	2.362	0.591	0.209
II(Kurt.)	-0.473	2.455	0.569	0.728	0.771	0.362	0.752	0.252	0.645	0.201
E(CDF)_out	0.044	0.043	0.025	0.030	0.025	0.025	0.024	0.030	0.006	0.017
E(CDF)_in	0.016	0.021	0.051	0.059	0.133	0.111	0.264	0.184	0.123	0.223

f) ε= 0 η= 0.25

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-1.829	3.020	4.103	2.983	9.856	2.856	15.035	2.455	18.870	1.627
Corr.Coeff.	0.941	0.047	0.941	0.047	0.937	0.047	0.925	0.049	0.882	0.054
β	0.802	0.107	0.802	0.107	0.803	0.107	0.804	0.106	0.804	0.105

γ	0.007	0.007	0.008	0.003	0.008	0.001	0.008	0.001	0.008	0.000
κ	0.259	0.071	0.262	0.071	0.271	0.069	0.306	0.064	0.410	0.050
II(RMS)	6.045	3.201	2.248	0.635	1.350	0.178	1.109	0.059	1.030	0.022
II(MM)	7.746	7.303	2.543	1.387	1.550	0.609	1.198	0.271	1.045	0.092
II(Skew.)	2.186	2.184	1.269	0.732	1.035	0.508	0.679	1.494	1.459	2.019
II(Kurt.)	2.918	2.138	1.576	0.617	1.255	0.289	1.129	0.203	0.926	0.153
E(CDF)_out	0.040	0.053	0.025	0.025	0.033	0.046	0.017	0.033	0.001	0.003
E(CDF)_in	0.016	0.021	0.051	0.059	0.133	0.111	0.264	0.184	0.123	0.223

g) ε= 0 η= 0.5

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-2.998	2.718	2.970	2.701	8.837	2.633	14.343	2.382	18.878	1.793
Corr.Coeff.	0.933	0.047	0.932	0.048	0.929	0.048	0.919	0.050	0.882	0.056
β	0.738	0.118	0.738	0.118	0.739	0.118	0.739	0.116	0.739	0.114
γ	0.006	0.006	0.007	0.003	0.007	0.001	0.007	0.001	0.006	0.000
κ	0.259	0.056	0.261	0.056	0.268	0.056	0.294	0.052	0.374	0.044
II(RMS)	8.132	4.228	2.765	0.792	1.499	0.217	1.161	0.071	1.054	0.026
II(MM)	9.985	10.457	2.986	1.743	1.697	0.721	1.261	0.319	1.078	0.113
II(Skew.)	1.941	3.189	1.151	1.244	0.922	0.984	0.304	2.996	1.905	3.639
II(Kurt.)	3.278	2.323	1.689	0.671	1.311	0.318	1.180	0.232	0.998	0.194
E(CDF)_out	0.045	0.054	0.030	0.028	0.041	0.058	0.023	0.037	0.001	0.001
E(CDF)_in	0.016	0.021	0.051	0.059	0.133	0.111	0.264	0.184	0.123	0.223

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h) ε= 0 η= 0.75

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-4.828	2.434	1.157	2.423	7.087	2.380	12.798	2.218	17.908	1.849
Corr.Coeff.	0.903	0.062	0.903	0.062	0.900	0.063	0.890	0.064	0.854	0.071
β	0.644	0.139	0.644	0.139	0.645	0.138	0.644	0.136	0.641	0.134
γ	0.005	0.004	0.005	0.002	0.005	0.001	0.005	0.001	0.004	0.001
κ	0.276	0.038	0.277	0.038	0.283	0.038	0.303	0.035	0.365	0.032
II(RMS)	10.838	5.458	3.439	0.969	1.694	0.263	1.228	0.087	1.082	0.033
II(MM)	12.798	14.686	3.556	2.273	1.903	0.914	1.348	0.401	1.118	0.140
II(Skew.)	0.845	5.221	0.759	2.236	0.649	1.847	-0.461	5.841	2.825	7.264
II(Kurt.)	3.482	2.836	1.770	0.851	1.356	0.410	1.220	0.312	1.048	0.276
E(CDF)_out	0.054	0.053	0.038	0.038	0.046	0.065	0.031	0.042	0.002	0.004
E(CDF)_in	0.016	0.021	0.051	0.059	0.133	0.111	0.264	0.184	0.123	0.223

i) ε= 0 η= 1

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-7.026	2.405	-1.042	2.389	4.883	2.333	10.590	2.155	15.698	1.782
Corr.Coeff.	0.805	0.172	0.805	0.171	0.801	0.168	0.784	0.165	0.724	0.160
β	0.522	0.173	0.522	0.172	0.522	0.171	0.519	0.168	0.512	0.167
γ	0.004	0.002	0.004	0.001	0.003	0.001	0.003	0.001	0.001	0.001
κ	0.330	0.068	0.332	0.068	0.338	0.066	0.363	0.060	0.444	0.048
II(RMS)	13.158	6.043	4.024	0.916	1.863	0.252	1.284	0.086	1.099	0.033
II(MM)	15.665	18.975	4.153	2.777	2.113	1.092	1.429	0.477	1.145	0.164
II(Skew.)	-2.368	8.676	-0.284	3.675	0.048	3.054	-1.753	10.034	3.998	12.685
II(Kurt.)	2.204	4.594	1.406	1.418	1.182	0.677	1.064	0.514	0.817	0.420
E(CDF)_out	0.065	0.047	0.045	0.049	0.058	0.068	0.039	0.049	0.002	0.005
E(CDF)_in	0.016	0.021	0.051	0.059	0.133	0.111	0.264	0.184	0.123	0.223

j) ε= 0.25 η= 0.25

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-4.273	1.630	1.706	1.628	7.610	1.595	13.292	1.514	18.157	1.240
Corr.Coeff.	0.917	0.040	0.917	0.041	0.914	0.041	0.903	0.043	0.865	0.047
β	0.704	0.080	0.704	0.080	0.704	0.080	0.706	0.079	0.711	0.078
γ	0.006	0.007	0.006	0.003	0.007	0.002	0.007	0.001	0.007	0.000
κ	0.293	0.041	0.294	0.041	0.301	0.041	0.323	0.041	0.401	0.037
II(RMS)	8.869	3.825	2.951	0.558	1.553	0.139	1.178	0.045	1.058	0.016
II(MM)	10.414	9.810	3.116	1.673	1.745	0.670	1.278	0.286	1.082	0.091
II(Skew.)	1.306	1.496	1.048	0.429	0.986	0.197	0.883	0.364	1.092	0.821
II(Kurt.)	1.862	1.902	1.247	0.592	1.095	0.276	1.020	0.193	0.861	0.146
E(CDF)_out	0.043	0.047	0.033	0.035	0.033	0.038	0.023	0.036	0.002	0.004
E(CDF)_in	0.016	0.021	0.051	0.059	0.133	0.111	0.264	0.184	0.123	0.223

k) ε= 0.5 η= 0.5

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-5.712	1.190	0.243	1.157	6.120	1.102	11.875	1.050	17.328	0.945
Corr.Coeff.	0.901	0.036	0.900	0.035	0.896	0.036	0.884	0.037	0.851	0.041
β	0.602	0.076	0.600	0.074	0.596	0.071	0.592	0.071	0.592	0.067
γ	0.004	0.006	0.005	0.003	0.005	0.001	0.005	0.001	0.005	0.000
κ	0.281	0.029	0.283	0.029	0.289	0.030	0.306	0.030	0.358	0.028
II(RMS)	12.242	5.315	3.795	0.690	1.802	0.156	1.268	0.050	1.098	0.017
II(MM)	13.793	14.180	3.839	2.158	1.999	0.832	1.391	0.356	1.137	0.108
II(Skew.)	0.727	1.685	0.849	0.554	0.867	0.333	0.680	0.926	1.107	1.015
II(Kurt.)	1.667	1.948	1.169	0.622	1.043	0.300	0.979	0.209	0.851	0.177

E(CDF)_out	0.054	0.051	0.043	0.045	0.048	0.053	0.032	0.040	0.002	0.003
E(CDF)_in	0.016	0.021	0.051	0.059	0.133	0.111	0.264	0.184	0.123	0.223

l) ε= 0.75 η= 0.75

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-7.607	0.920	-1.661	0.884	4.203	0.911	9.989	0.834	15.681	0.738
Corr.Coeff.	0.857	0.036	0.854	0.036	0.844	0.039	0.828	0.039	0.796	0.041
β	0.459	0.081	0.457	0.079	0.449	0.083	0.439	0.081	0.430	0.074
γ	0.003	0.004	0.003	0.002	0.003	0.001	0.003	0.001	0.002	0.001
κ	0.269	0.027	0.272	0.027	0.278	0.030	0.290	0.033	0.321	0.031
II(RMS)	16.717	7.462	4.904	0.913	2.127	0.214	1.382	0.067	1.149	0.024
II(MM)	18.042	19.420	4.776	2.779	2.357	1.036	1.546	0.452	1.219	0.142
II(Skew.)	-1.024	2.956	0.262	1.243	0.488	1.036	-0.058	3.223	1.862	4.388
II(Kurt.)	1.300	2.537	1.035	0.843	0.942	0.414	0.885	0.316	0.789	0.252
E(CDF)_out	0.068	0.058	0.062	0.065	0.069	0.073	0.043	0.050	0.003	0.005
E(CDF)_in	0.016	0.021	0.051	0.059	0.133	0.111	0.264	0.184	0.123	0.223

m) ε= 1 η= 1

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-9.562	1.129	-3.557	1.140	2.247	1.107	7.946	0.960	13.556	0.797
Corr.Coeff.	0.697	0.180	0.697	0.174	0.665	0.188	0.621	0.184	0.542	0.152
β	0.294	0.106	0.296	0.107	0.284	0.109	0.265	0.101	0.251	0.095
γ	0.002	0.002	0.001	0.001	0.001	0.001	0.000	0.001	-0.001	0.001
κ	0.280	0.058	0.284	0.058	0.297	0.064	0.316	0.070	0.371	0.055
II(RMS)	20.651	9.450	5.843	1.027	2.384	0.222	1.468	0.071	1.175	0.025
II(MM)	23.031	25.527	5.882	3.403	2.723	1.199	1.710	0.494	1.288	0.158
II(Skew.)	-5.642	6.428	-1.184	2.474	-0.305	2.061	-1.508	7.324	2.932	10.018
II(Kurt.)	-0.878	4.294	0.338	1.307	0.566	0.645	0.539	0.397	0.361	0.279
E(CDF)_out	0.082	0.060	0.078	0.086	0.086	0.088	0.050	0.058	0.005	0.011
E(CDF)_in	0.016	0.021	0.051	0.059	0.133	0.111	0.264	0.184	0.123	0.223

Table 5.3: Results for ECG with EMG noise

a) ε= 0 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-1.594	2.547	1.869	1.585	3.630	0.670	4.277	0.208	4.466	0.056
Corr.Coeff.	0.944	0.040	0.912	0.041	0.811	0.042	0.599	0.038	0.358	0.026
β	0.913	0.073	0.913	0.073	0.913	0.072	0.912	0.071	0.911	0.071
γ	0.350	0.022	0.351	0.011	0.352	0.006	0.352	0.003	0.352	0.001
κ	0.286	0.084	0.356	0.070	0.548	0.047	0.991	0.027	1.921	0.014
II(RMS)	2.916	3.490	1.022	0.383	0.691	0.077	0.571	0.017	0.498	0.005
II(MM)	0.630	1.450	0.468	0.280	0.357	0.134	0.238	0.146	0.181	0.081
II(Skew.)	1.242	1.282	0.705	0.357	0.365	0.191	-0.068	0.336	-0.384	0.516
II(Kurt.)	4.946	9.923	1.068	0.594	0.533	0.211	0.274	0.241	0.471	0.675
E(CDF)_out	0.017	0.038	0.015	0.031	0.024	0.023	0.070	0.066	0.091	0.106
E(CDF)_in	0.011	0.017	0.043	0.054	0.083	0.069	0.156	0.142	0.153	0.127

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b) ε= 0.25 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-3.545	1.525	1.332	1.287	4.155	0.750	5.095	0.256	5.364	0.056
Corr.Coeff.	0.921	0.035	0.898	0.037	0.809	0.039	0.595	0.034	0.335	0.018
β	0.823	0.054	0.826	0.054	0.826	0.055	0.811	0.053	0.760	0.044
γ	0.199	0.047	0.241	0.032	0.275	0.016	0.294	0.005	0.299	0.002
κ	0.336	0.054	0.378	0.051	0.529	0.041	0.923	0.024	1.771	0.009
II(RMS)	5.966	4.034	1.770	0.336	0.951	0.058	0.702	0.011	0.590	0.003
II(MM)	1.510	1.382	0.751	0.345	0.471	0.128	0.280	0.153	0.187	0.082
II(Skew.)	0.267	2.185	0.533	0.512	0.342	0.274	-0.103	0.399	-0.462	0.595
II(Kurt.)	4.299	10.898	0.990	0.693	0.570	0.267	0.407	0.367	0.893	1.204
E(CDF)_out	0.022	0.041	0.017	0.027	0.020	0.015	0.054	0.054	0.077	0.098
E(CDF)_in	0.011	0.017	0.043	0.054	0.083	0.069	0.156	0.142	0.153	0.127

c) ε= 0.5 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-4.559	1.078	0.631	0.959	4.082	0.640	5.419	0.237	5.826	0.061
Corr.Coeff.	0.903	0.031	0.881	0.032	0.799	0.034	0.592	0.029	0.337	0.018
β	0.778	0.045	0.781	0.045	0.784	0.046	0.768	0.043	0.722	0.042
γ	0.180	0.043	0.215	0.029	0.245	0.016	0.263	0.006	0.269	0.002
κ	0.362	0.041	0.399	0.040	0.534	0.034	0.902	0.020	1.709	0.009
II(RMS)	7.278	4.310	2.072	0.345	1.049	0.059	0.756	0.013	0.630	0.004
II(MM)	1.565	1.302	0.859	0.355	0.561	0.139	0.346	0.183	0.198	0.080

II(Skew.)	-0.276	2.884	0.377	0.613	0.311	0.314	-0.108	0.419	-0.477	0.625
II(Kurt.)	3.305	9.671	0.832	0.710	0.538	0.271	0.402	0.352	0.955	1.261
E(CDF)_out	0.025	0.040	0.021	0.027	0.021	0.014	0.050	0.053	0.073	0.098
E(CDF)_in	0.011	0.017	0.043	0.054	0.083	0.069	0.156	0.142	0.153	0.127

d) $\varepsilon = 0.75$ $\eta = 0$

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-5.663	0.808	-0.213	0.703	3.897	0.495	5.762	0.224	6.357	0.068
Corr.Coeff.	0.875	0.027	0.856	0.027	0.783	0.029	0.587	0.027	0.337	0.022
β	0.725	0.046	0.729	0.046	0.733	0.045	0.718	0.043	0.676	0.048
γ	0.156	0.038	0.184	0.025	0.208	0.017	0.227	0.007	0.235	0.002
κ	0.396	0.030	0.427	0.027	0.541	0.025	0.878	0.017	1.639	0.008
II(RMS)	8.633	4.690	2.398	0.401	1.164	0.081	0.817	0.019	0.674	0.005
II(MM)	1.837	1.276	1.006	0.377	0.697	0.171	0.441	0.176	0.223	0.082
II(Skew.)	-1.013	3.746	0.209	0.744	0.285	0.347	-0.101	0.434	-0.489	0.646
II(Kurt.)	2.021	8.057	0.632	0.651	0.500	0.262	0.396	0.350	1.060	1.398
E(CDF)_out	0.028	0.042	0.024	0.025	0.022	0.013	0.045	0.049	0.069	0.097
E(CDF)_in	0.011	0.017	0.043	0.054	0.083	0.069	0.156	0.142	0.153	0.127

e) $\varepsilon = 1$ $\eta = 0$

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-6.399	0.742	-0.791	0.626	3.681	0.414	5.896	0.214	6.629	0.069
Corr.Coeff.	0.850	0.027	0.834	0.027	0.768	0.027	0.581	0.029	0.335	0.026
β	0.683	0.053	0.690	0.052	0.698	0.048	0.688	0.048	0.647	0.055
γ	0.138	0.033	0.164	0.024	0.190	0.018	0.210	0.008	0.217	0.002
κ	0.417	0.023	0.445	0.020	0.548	0.019	0.866	0.016	1.603	0.006
II(RMS)	9.703	4.937	2.625	0.459	1.231	0.096	0.849	0.025	0.697	0.006
II(MM)	1.985	1.442	1.091	0.447	0.771	0.131	0.494	0.151	0.272	0.079
II(Skew.)	-2.021	4.274	0.015	0.868	0.238	0.372	-0.107	0.420	-0.493	0.645
II(Kurt.)	1.281	8.497	0.495	0.688	0.452	0.249	0.365	0.300	1.072	1.416
E(CDF)_out	0.032	0.043	0.027	0.026	0.023	0.013	0.041	0.046	0.067	0.095
E(CDF)_in	0.011	0.017	0.043	0.054	0.083	0.069	0.156	0.142	0.153	0.127

f) $\varepsilon = 0$ $\eta = 0.25$

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-2.733	2.538	1.431	1.771	3.930	0.831	5.163	0.255	5.786	0.091
Corr.Coeff.	0.929	0.048	0.895	0.051	0.795	0.057	0.596	0.055	0.367	0.040
β	0.803	0.107	0.805	0.107	0.807	0.107	0.809	0.105	0.805	0.103

γ	0.325	0.020	0.322	0.012	0.315	0.009	0.304	0.007	0.293	0.004
κ	0.283	0.066	0.345	0.054	0.514	0.035	0.894	0.019	1.667	0.019
II(RMS)	7.731	7.849	2.002	0.799	0.975	0.172	0.710	0.048	0.615	0.016
II(MM)	3.209	2.621	1.263	0.969	0.733	0.326	0.542	0.098	0.489	0.053
II(Skew.)	1.718	2.404	0.805	0.453	0.397	0.136	-0.006	0.266	-0.291	0.416
II(Kurt.)	2.809	2.612	1.017	0.612	0.494	0.174	0.185	0.138	0.193	0.314
E(CDF)_out	0.027	0.044	0.019	0.028	0.019	0.020	0.052	0.053	0.074	0.098
E(CDF)_in	0.011	0.017	0.043	0.054	0.083	0.069	0.156	0.142	0.153	0.127

g) ε= 0 η= 0.5

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-3.649	2.363	0.908	1.743	3.875	0.860	5.476	0.291	6.368	0.167
Corr.Coeff.	0.919	0.050	0.883	0.054	0.780	0.062	0.585	0.060	0.362	0.044
β	0.739	0.118	0.742	0.118	0.747	0.117	0.749	0.115	0.738	0.112
γ	0.312	0.019	0.308	0.013	0.298	0.013	0.284	0.011	0.267	0.007
κ	0.282	0.051	0.340	0.042	0.501	0.026	0.857	0.026	1.565	0.037
II(RMS)	10.438	9.991	2.554	0.977	1.125	0.215	0.774	0.065	0.665	0.024
II(MM)	4.071	3.395	1.525	1.143	0.862	0.405	0.631	0.112	0.568	0.050
II(Skew.)	1.441	4.054	0.731	0.670	0.364	0.148	-0.005	0.241	-0.274	0.393
II(Kurt.)	3.291	2.762	1.057	0.650	0.491	0.186	0.184	0.133	0.173	0.294
E(CDF)_out	0.034	0.052	0.020	0.025	0.021	0.021	0.044	0.048	0.066	0.091
E(CDF)_in	0.011	0.017	0.043	0.054	0.083	0.069	0.156	0.142	0.153	0.127

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h) ε= 0 η= 0.75

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-5.223	2.192	-0.192	1.733	3.475	0.932	5.704	0.365	7.051	0.296
Corr.Coeff.	0.889	0.065	0.850	0.072	0.744	0.078	0.555	0.071	0.340	0.053
β	0.646	0.138	0.650	0.137	0.657	0.136	0.659	0.132	0.629	0.127
γ	0.295	0.020	0.290	0.017	0.277	0.019	0.257	0.018	0.234	0.011
κ	0.296	0.034	0.349	0.028	0.496	0.024	0.822	0.045	1.452	0.067
II(RMS)	14.123	13.435	3.283	1.248	1.321	0.276	0.856	0.090	0.728	0.036
II(MM)	5.170	4.536	1.857	1.412	1.017	0.507	0.731	0.137	0.657	0.050
II(Skew.)	0.302	6.890	0.448	1.075	0.260	0.213	-0.032	0.221	-0.265	0.369
II(Kurt.)	3.995	3.952	1.051	0.771	0.466	0.215	0.183	0.140	0.173	0.299
E(CDF)_out	0.043	0.069	0.030	0.036	0.028	0.026	0.035	0.044	0.058	0.088
E(CDF)_in	0.011	0.017	0.043	0.054	0.083	0.069	0.156	0.142	0.153	0.127

i) ε= 0 η= 1

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-7.244	2.236	-1.838	1.900	2.545	1.170	5.598	0.499	7.606	0.461
Corr.Coeff.	0.793	0.165	0.756	0.156	0.658	0.135	0.488	0.101	0.282	0.073
β	0.525	0.170	0.530	0.168	0.537	0.165	0.535	0.158	0.469	0.152
γ	0.275	0.024	0.268	0.024	0.252	0.028	0.224	0.027	0.192	0.018
κ	0.347	0.064	0.392	0.056	0.521	0.046	0.811	0.068	1.365	0.107
II(RMS)	17.507	16.388	3.956	1.436	1.507	0.319	0.939	0.115	0.794	0.051
II(MM)	6.377	5.831	2.231	1.728	1.180	0.586	0.843	0.179	0.741	0.049
II(Skew.)	-3.573	10.227	-0.401	1.602	0.002	0.344	-0.107	0.229	-0.274	0.354
II(Kurt.)	-0.181	11.537	0.653	1.160	0.331	0.281	0.135	0.114	0.129	0.224
E(CDF)_out	0.052	0.079	0.061	0.066	0.033	0.028	0.028	0.044	0.053	0.093
E(CDF)_in	0.011	0.017	0.043	0.054	0.083	0.069	0.156	0.142	0.153	0.127

j) ε= 0.25 η= 0.25

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-4.375	1.652	0.896	1.480	4.506	0.919	6.253	0.282	7.198	0.161
Corr.Coeff.	0.912	0.042	0.889	0.046	0.802	0.052	0.605	0.048	0.356	0.028
β	0.709	0.083	0.714	0.084	0.717	0.085	0.706	0.083	0.651	0.063
γ	0.174	0.045	0.211	0.028	0.236	0.012	0.243	0.005	0.234	0.005
κ	0.302	0.041	0.340	0.039	0.468	0.028	0.785	0.017	1.423	0.031
II(RMS)	11.330	8.657	2.869	0.724	1.275	0.140	0.865	0.042	0.733	0.016
II(MM)	4.437	3.149	1.590	1.023	0.857	0.317	0.614	0.104	0.538	0.046
II(Skew.)	0.898	1.429	0.699	0.371	0.405	0.176	-0.023	0.323	-0.359	0.488
II(Kurt.)	1.523	3.618	0.978	0.664	0.545	0.198	0.292	0.212	0.541	0.746
E(CDF)_out	0.034	0.049	0.025	0.033	0.017	0.017	0.033	0.038	0.057	0.084
E(CDF)_in	0.011	0.017	0.043	0.054	0.083	0.069	0.156	0.142	0.153	0.127

k) ε= 0.5 η= 0.5

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-5.887	1.196	-0.257	1.120	4.277	0.806	7.003	0.317	8.618	0.266
Corr.Coeff.	0.893	0.038	0.869	0.042	0.787	0.049	0.601	0.045	0.362	0.029
β	0.598	0.078	0.605	0.079	0.613	0.080	0.602	0.077	0.547	0.058
γ	0.141	0.037	0.171	0.023	0.191	0.010	0.193	0.008	0.182	0.007
κ	0.291	0.027	0.325	0.025	0.433	0.019	0.695	0.030	1.203	0.049
II(RMS)	15.946	11.256	3.850	0.862	1.563	0.166	1.002	0.053	0.836	0.022
II(MM)	6.016	4.118	2.071	1.319	1.087	0.422	0.791	0.121	0.664	0.046
II(Skew.)	0.105	1.775	0.497	0.403	0.359	0.173	-0.019	0.321	-0.363	0.508
II(Kurt.)	0.522	5.731	0.888	0.673	0.525	0.183	0.286	0.189	0.585	0.814
E(CDF)_out	0.044	0.059	0.039	0.042	0.028	0.027	0.019	0.030	0.042	0.072

E(CDF)_in	0.011	0.017	0.043	0.054	0.083	0.069	0.156	0.142	0.153	0.127

l) ε= 0.75 η= 0.75

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-7.745	0.980	-1.898	0.926	3.448	0.732	7.461	0.356	10.219	0.356
Corr.Coeff.	0.845	0.039	0.820	0.044	0.745	0.053	0.575	0.051	0.345	0.041
β	0.454	0.088	0.459	0.088	0.469	0.085	0.458	0.080	0.393	0.064
γ	0.100	0.029	0.123	0.016	0.136	0.010	0.136	0.010	0.122	0.009
κ	0.278	0.028	0.305	0.028	0.387	0.027	0.584	0.048	0.948	0.067
II(RMS)	21.922	15.673	5.132	1.166	1.942	0.217	1.172	0.070	0.956	0.028
II(MM)	8.074	6.097	2.713	1.724	1.370	0.564	0.981	0.164	0.802	0.049
II(Skew.)	-1.762	3.193	0.055	0.680	0.247	0.195	-0.026	0.295	-0.337	0.477
II(Kurt.)	-0.182	6.592	0.788	0.781	0.537	0.229	0.375	0.269	0.923	1.320
E(CDF)_out	0.064	0.082	0.063	0.053	0.046	0.045	0.016	0.025	0.031	0.060
E(CDF)_in	0.011	0.017	0.043	0.054	0.083	0.069	0.156	0.142	0.153	0.127

m) ε= 1 η= 1

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-9.679	1.174	-3.682	1.184	2.043	0.976	6.908	0.603	10.388	0.385
Corr.Coeff.	0.673	0.175	0.659	0.169	0.603	0.143	0.458	0.112	0.213	0.088
β	0.288	0.113	0.298	0.115	0.310	0.106	0.302	0.099	0.200	0.089
γ	0.062	0.021	0.082	0.015	0.094	0.012	0.091	0.015	0.072	0.013
κ	0.292	0.060	0.312	0.058	0.377	0.051	0.535	0.047	0.848	0.057
II(RMS)	27.251	19.900	6.209	1.466	2.249	0.253	1.292	0.071	1.025	0.025
II(MM)	10.369	7.527	3.275	2.081	1.609	0.640	1.081	0.174	0.877	0.053
II(Skew.)	-8.040	7.454	-1.310	1.349	-0.169	0.371	-0.145	0.283	-0.329	0.413
II(Kurt.)	-6.457	21.631	0.140	1.576	0.359	0.412	0.355	0.332	0.599	0.721
E(CDF)_out	0.078	0.084	0.086	0.085	0.057	0.047	0.029	0.038	0.033	0.073
E(CDF)_in	0.011	0.017	0.043	0.054	0.083	0.069	0.156	0.142	0.153	0.127

Table 5.4: Results for ECG with MA

a) ε= 0 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-1.231	2.719	2.646	1.839	4.787	0.854	5.634	0.282	5.892	0.077
Corr.Coeff.	0.947	0.040	0.923	0.040	0.845	0.043	0.657	0.043	0.409	0.035
β	0.913	0.074	0.912	0.074	0.911	0.075	0.909	0.078	0.905	0.085
γ	0.244	0.034	0.248	0.017	0.250	0.008	0.251	0.004	0.252	0.002
κ	0.281	0.086	0.341	0.073	0.510	0.051	0.908	0.029	1.750	0.016
II(RMS)	2.358	1.664	1.117	0.342	0.812	0.090	0.690	0.026	0.609	0.008
II(MM)	1.090	5.823	1.172	1.573	0.846	0.725	0.577	0.117	0.411	0.069
II(Skew.)	1.378	1.093	0.865	0.330	0.632	0.208	0.415	0.358	0.316	0.570
II(Kurt.)	2.435	1.622	1.133	0.403	0.672	0.131	0.362	0.123	0.311	0.209
E(CDF)_out	0.023	0.031	0.008	0.015	0.033	0.032	0.134	0.116	0.131	0.096
E(CDF)_in	0.019	0.018	0.032	0.041	0.139	0.102	0.298	0.144	0.342	0.179

b) ε= 0.25 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-3.727	1.435	1.203	1.160	4.459	0.676	5.765	0.278	6.075	0.091
Corr.Coeff.	0.918	0.034	0.895	0.035	0.819	0.036	0.631	0.037	0.378	0.030
β	0.818	0.054	0.820	0.055	0.823	0.055	0.825	0.060	0.805	0.069
γ	0.214	0.036	0.219	0.018	0.226	0.009	0.235	0.005	0.241	0.003
κ	0.341	0.052	0.385	0.046	0.525	0.036	0.896	0.024	1.717	0.014
II(RMS)	4.939	1.693	1.748	0.275	0.992	0.062	0.751	0.018	0.637	0.006
II(MM)	2.465	3.966	1.423	1.405	0.935	0.643	0.640	0.135	0.450	0.063
II(Skew.)	0.074	1.823	0.548	0.409	0.531	0.113	0.371	0.298	0.300	0.563
II(Kurt.)	1.234	1.914	0.838	0.489	0.580	0.149	0.332	0.119	0.316	0.217
E(CDF)_out	0.024	0.029	0.014	0.017	0.033	0.035	0.127	0.109	0.125	0.095
E(CDF)_in	0.019	0.018	0.032	0.041	0.139	0.102	0.298	0.144	0.342	0.179

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c) ε= 0.5 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-4.540	1.069	0.530	0.889	4.070	0.543	5.662	0.238	6.091	0.078
Corr.Coeff.	0.903	0.030	0.879	0.031	0.798	0.031	0.609	0.031	0.363	0.026
β	0.781	0.046	0.780	0.045	0.779	0.045	0.781	0.050	0.764	0.061
γ	0.214	0.036	0.219	0.019	0.224	0.009	0.232	0.005	0.238	0.003
κ	0.362	0.041	0.404	0.037	0.541	0.029	0.904	0.020	1.716	0.012
II(RMS)	5.885	1.795	1.985	0.266	1.059	0.056	0.770	0.016	0.644	0.006

II(MM)	2.944	3.213	1.506	1.346	0.970	0.621	0.656	0.134	0.465	0.069
II(Skew.)	-0.581	2.574	0.380	0.547	0.453	0.114	0.340	0.280	0.290	0.559
II(Kurt.)	0.661	2.270	0.675	0.542	0.504	0.171	0.306	0.123	0.309	0.218
E(CDF)_out	0.027	0.029	0.015	0.017	0.037	0.038	0.126	0.108	0.125	0.096
E(CDF)_in	0.019	0.018	0.032	0.041	0.139	0.102	0.298	0.144	0.342	0.179

d) ε= 0.75 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-5.636	0.812	-0.383	0.696	3.519	0.432	5.494	0.191	6.108	0.063
Corr.Coeff.	0.876	0.027	0.850	0.027	0.765	0.028	0.576	0.030	0.342	0.025
β	0.729	0.045	0.728	0.045	0.723	0.045	0.721	0.050	0.710	0.058
γ	0.214	0.036	0.217	0.018	0.221	0.009	0.228	0.005	0.233	0.003
κ	0.395	0.030	0.435	0.028	0.565	0.021	0.916	0.014	1.712	0.009
II(RMS)	7.031	2.140	2.261	0.329	1.136	0.072	0.795	0.020	0.654	0.006
II(MM)	3.109	3.215	1.576	1.361	1.016	0.617	0.679	0.136	0.479	0.072
II(Skew.)	-1.605	3.540	0.118	0.706	0.356	0.141	0.302	0.283	0.278	0.546
II(Kurt.)	-0.094	2.666	0.463	0.622	0.427	0.191	0.279	0.129	0.304	0.215
E(CDF)_out	0.030	0.029	0.018	0.018	0.040	0.040	0.125	0.106	0.124	0.096
E(CDF)_in	0.019	0.018	0.032	0.041	0.139	0.102	0.298	0.144	0.342	0.179

e) ε= 1 η= 0

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-6.418	0.787	-1.058	0.676	3.092	0.431	5.362	0.184	6.115	0.052
Corr.Coeff.	0.849	0.030	0.822	0.031	0.735	0.033	0.549	0.035	0.324	0.027
β	0.685	0.054	0.683	0.053	0.679	0.056	0.675	0.058	0.665	0.062
γ	0.212	0.036	0.215	0.018	0.219	0.009	0.224	0.004	0.230	0.003
κ	0.419	0.026	0.457	0.024	0.582	0.016	0.922	0.010	1.710	0.007
II(RMS)	7.992	2.529	2.485	0.408	1.193	0.095	0.814	0.024	0.661	0.007
II(MM)	3.072	3.057	1.577	1.442	1.080	0.577	0.710	0.137	0.493	0.072
II(Skew.)	-2.323	3.704	-0.140	0.831	0.276	0.159	0.270	0.273	0.264	0.535
II(Kurt.)	-0.946	2.914	0.252	0.685	0.358	0.200	0.258	0.137	0.296	0.213
E(CDF)_out	0.032	0.030	0.021	0.019	0.042	0.042	0.125	0.104	0.123	0.096
E(CDF)_in	0.019	0.018	0.032	0.041	0.139	0.102	0.298	0.144	0.342	0.179

f) ε= 0 η= 0.25

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-2.275	2.764	2.545	2.208	5.784	1.308	7.483	0.569	8.237	0.206
Corr.Coeff.	0.936	0.047	0.917	0.050	0.853	0.056	0.694	0.061	0.461	0.051
β	0.802	0.107	0.802	0.107	0.800	0.105	0.795	0.100	0.796	0.101
γ	0.179	0.026	0.186	0.014	0.189	0.008	0.186	0.005	0.182	0.003
κ	0.272	0.068	0.312	0.060	0.434	0.047	0.725	0.034	1.342	0.025
II(RMS)	6.121	3.414	2.103	0.636	1.160	0.160	0.884	0.041	0.767	0.013
II(MM)	7.462	10.368	2.833	4.409	1.610	2.089	0.897	0.222	0.715	0.065
II(Skew.)	2.196	2.330	1.077	0.650	0.748	0.340	0.484	0.367	0.308	0.490
II(Kurt.)	2.861	2.095	1.284	0.553	0.781	0.213	0.410	0.115	0.235	0.136
E(CDF)_out	0.037	0.038	0.022	0.032	0.019	0.013	0.083	0.084	0.081	0.072
E(CDF)_in	0.019	0.018	0.032	0.041	0.139	0.102	0.298	0.144	0.342	0.179

g) ε= 0 η= 0.5

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-3.272	2.569	1.910	2.164	5.758	1.390	8.056	0.682	9.253	0.302
Corr.Coeff.	0.928	0.048	0.910	0.051	0.848	0.059	0.696	0.066	0.472	0.056
β	0.738	0.118	0.738	0.118	0.735	0.112	0.726	0.101	0.721	0.100
γ	0.160	0.023	0.168	0.013	0.170	0.008	0.165	0.006	0.158	0.004
κ	0.269	0.054	0.303	0.049	0.409	0.039	0.661	0.034	1.184	0.034
II(RMS)	8.252	4.074	2.644	0.745	1.331	0.185	0.968	0.045	0.830	0.016
II(MM)	9.718	14.067	3.429	5.154	1.888	2.437	1.013	0.252	0.801	0.069
II(Skew.)	2.203	3.585	1.055	0.993	0.748	0.464	0.495	0.412	0.305	0.478
II(Kurt.)	3.235	2.273	1.397	0.609	0.833	0.248	0.442	0.133	0.226	0.122
E(CDF)_out	0.046	0.042	0.028	0.038	0.020	0.021	0.063	0.065	0.062	0.062
E(CDF)_in	0.019	0.018	0.032	0.041	0.139	0.102	0.298	0.144	0.342	0.179

h) ε= 0 η= 0.75

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-4.958	2.381	0.579	2.122	5.188	1.471	8.441	0.757	10.489	0.362
Corr.Coeff.	0.899	0.062	0.882	0.063	0.825	0.067	0.683	0.072	0.469	0.065
β	0.644	0.140	0.644	0.138	0.639	0.127	0.621	0.107	0.598	0.107
γ	0.137	0.022	0.142	0.014	0.143	0.010	0.134	0.008	0.124	0.007
κ	0.283	0.038	0.309	0.034	0.392	0.026	0.593	0.026	1.001	0.035
II(RMS)	11.076	5.016	3.364	0.900	1.560	0.225	1.078	0.055	0.913	0.022
II(MM)	12.623	19.557	4.147	6.233	2.235	2.959	1.151	0.302	0.912	0.078
II(Skew.)	1.570	5.498	0.844	1.581	0.689	0.671	0.481	0.485	0.278	0.466

II(Kurt.)	3.583	2.891	1.507	0.778	0.892	0.321	0.487	0.177	0.215	0.111
E(CDF)_out	0.053	0.052	0.037	0.048	0.029	0.037	0.039	0.038	0.040	0.056
E(CDF)_in	0.019	0.018	0.032	0.041	0.139	0.102	0.298	0.144	0.342	0.179

i) ε= 0 η= 1

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-7.086	2.398	-1.342	2.230	3.820	1.673	7.850	0.890	10.666	0.513
Corr.Coeff.	0.800	0.168	0.786	0.158	0.740	0.124	0.601	0.098	0.377	0.097
β	0.522	0.173	0.520	0.170	0.512	0.152	0.478	0.122	0.418	0.131
γ	0.107	0.027	0.110	0.022	0.107	0.015	0.095	0.012	0.080	0.010
κ	0.335	0.065	0.353	0.057	0.415	0.040	0.584	0.035	0.952	0.056
II(RMS)	13.783	6.125	4.040	0.980	1.781	0.242	1.175	0.063	0.969	0.025
II(MM)	15.506	24.775	4.873	7.407	2.569	3.499	1.271	0.355	1.003	0.104
II(Skew.)	-1.977	7.120	-0.141	2.397	0.356	0.953	0.293	0.540	0.075	0.379
II(Kurt.)	2.597	4.713	1.190	1.383	0.792	0.539	0.426	0.246	0.119	0.092
E(CDF)_out	0.075	0.075	0.048	0.059	0.042	0.051	0.036	0.035	0.034	0.071
E(CDF)_in	0.019	0.018	0.032	0.041	0.139	0.102	0.298	0.144	0.342	0.179

j) ε= 0.25 η= 0.25

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-4.427	1.587	1.049	1.429	5.348	1.050	7.705	0.582	8.596	0.239
Corr.Coeff.	0.912	0.041	0.896	0.043	0.835	0.049	0.675	0.056	0.431	0.047
β	0.705	0.081	0.706	0.081	0.708	0.079	0.708	0.077	0.693	0.082
γ	0.148	0.028	0.157	0.015	0.164	0.008	0.170	0.006	0.171	0.003
κ	0.301	0.039	0.329	0.037	0.426	0.034	0.689	0.032	1.277	0.026
II(RMS)	9.117	3.279	2.842	0.501	1.375	0.116	0.962	0.027	0.805	0.009
II(MM)	9.608	12.289	3.379	4.665	1.821	2.164	0.982	0.240	0.755	0.067
II(Skew.)	0.999	1.213	0.803	0.348	0.665	0.181	0.439	0.284	0.281	0.476
II(Kurt.)	1.652	1.700	0.996	0.472	0.699	0.174	0.375	0.092	0.231	0.141
E(CDF)_out	0.038	0.047	0.030	0.041	0.023	0.019	0.074	0.076	0.072	0.070
E(CDF)_in	0.019	0.018	0.032	0.041	0.139	0.102	0.298	0.144	0.342	0.179

k) ε= 0.5 η= 0.5

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-5.778	1.200	-0.124	1.106	4.760	0.858	8.085	0.578	9.717	0.322
Corr.Coeff.	0.897	0.037	0.879	0.040	0.816	0.046	0.659	0.052	0.425	0.045

β	0.602	0.078	0.601	0.077	0.599	0.071	0.595	0.061	0.579	0.068
γ	0.130	0.024	0.137	0.013	0.143	0.008	0.145	0.007	0.143	0.005
κ	0.287	0.028	0.310	0.027	0.392	0.027	0.610	0.033	1.090	0.036
II(RMS)	12.594	3.848	3.740	0.519	1.651	0.116	1.080	0.028	0.884	0.013
II(MM)	13.278	18.034	4.287	5.705	2.233	2.652	1.147	0.278	0.873	0.085
II(Skew.)	0.381	1.568	0.629	0.422	0.585	0.203	0.407	0.283	0.253	0.444
II(Kurt.)	1.272	1.807	0.899	0.482	0.650	0.188	0.361	0.093	0.197	0.113
E(CDF)_out	0.051	0.055	0.039	0.049	0.030	0.028	0.052	0.053	0.050	0.063
E(CDF)_in	0.019	0.018	0.032	0.041	0.139	0.102	0.298	0.144	0.342	0.179

l) ε= 0.75 η= 0.75

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-7.671	0.930	-1.861	0.890	3.552	0.661	8.022	0.422	10.977	0.329
Corr.Coeff.	0.852	0.035	0.833	0.037	0.769	0.041	0.614	0.045	0.386	0.048
β	0.457	0.084	0.453	0.084	0.446	0.071	0.429	0.051	0.400	0.062
γ	0.104	0.022	0.109	0.014	0.112	0.009	0.109	0.009	0.104	0.007
κ	0.274	0.030	0.289	0.031	0.347	0.029	0.503	0.034	0.855	0.041
II(RMS)	17.273	4.871	4.938	0.619	2.026	0.139	1.246	0.037	0.990	0.018
II(MM)	17.880	25.554	5.482	7.349	2.772	3.401	1.335	0.338	1.017	0.086
II(Skew.)	-1.309	2.648	0.163	0.734	0.427	0.302	0.333	0.312	0.189	0.378
II(Kurt.)	0.674	2.862	0.763	0.678	0.608	0.254	0.351	0.135	0.143	0.075
E(CDF)_out	0.075	0.071	0.056	0.065	0.047	0.046	0.031	0.032	0.029	0.063
E(CDF)_in	0.019	0.018	0.032	0.041	0.139	0.102	0.298	0.144	0.342	0.179

m) ε= 1 η= 1

Perform. index	SNR = 12 dB		SNR = 6 dB		SNR = 0 dB		SNR =-6 dB		SNR = -12 dB	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
SNR-impr.	-9.581	1.191	-3.747	1.108	1.925	0.804	6.983	0.506	10.670	0.534
Corr.Coeff.	0.682	0.181	0.656	0.178	0.608	0.123	0.447	0.098	0.211	0.098
β	0.295	0.116	0.284	0.110	0.271	0.086	0.241	0.070	0.181	0.087
γ	0.072	0.027	0.074	0.020	0.073	0.014	0.067	0.012	0.057	0.010
κ	0.287	0.050	0.295	0.040	0.327	0.032	0.450	0.025	0.796	0.056
II(RMS)	21.429	6.572	6.008	0.748	2.372	0.138	1.381	0.037	1.048	0.016
II(MM)	22.716	32.423	6.739	8.675	3.317	4.110	1.532	0.419	1.132	0.103
II(Skew.)	-6.977	6.646	-1.377	1.699	-0.101	0.543	0.045	0.290	-0.051	0.220
II(Kurt.)	-1.734	4.850	0.042	1.286	0.359	0.459	0.205	0.190	0.015	0.048
E(CDF)_out	0.099	0.107	0.077	0.091	0.066	0.072	0.044	0.039	0.039	0.078
E(CDF)_in	0.019	0.018	0.032	0.041	0.139	0.102	0.298	0.144	0.342	0.179

Table 5.5: Insertion and detection errors in R-peak detection for the BW-corrupted segment from ECG-105

a**)** Noisy ECG (No. of R-peaks = 82)

Temporal Tolerance (ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.012	0.012	0.073	0.183	0.329	0.012	0.012	0.073	0.171	0.317
10	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
30	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
40	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
50	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
60	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
70	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
80	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
90	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
100	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000

b) Denoised ECG with ε= 0 and η = 0

Temporal Tolerance e(ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.012	0.012	0.073	0.183	0.329	0.012	0.012	0.073	0.171	0.317
10	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
30	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
40	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
50	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
60	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
70	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
80	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
90	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000
100	0.000	0.000	0.000	0.012	0.012	0.000	0.000	0.000	0.000	0.000

Table 5.6: Insertion and detection errors in R-peak detection for the EMG-corrupted segment from ECG-105

a) Noisy ECG record: 105 (No. of R-peaks = 82)

Temporal Tolerance (ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.073	0.305	0.805	0.585	0.159	0.073	0.268	0.439	0.939	1.000
10	0.000	0.037	0.390	0.427	0.146	0.000	0.000	0.024	0.780	0.988
20	0.000	0.037	0.366	0.366	0.134	0.000	0.000	0.000	0.720	0.976
30	0.000	0.037	0.366	0.354	0.134	0.000	0.000	0.000	0.707	0.976
40	0.000	0.037	0.366	0.354	0.134	0.000	0.000	0.000	0.707	0.976
50	0.000	0.037	0.366	0.341	0.134	0.000	0.000	0.000	0.695	0.976

60	0.000	0.037	0.366	0.341	0.134	0.000	0.000	0.000	0.695	0.976
70	0.000	0.037	0.366	0.341	0.134	0.000	0.000	0.000	0.695	0.976
80	0.000	0.037	0.366	0.341	0.134	0.000	0.000	0.000	0.695	0.976
90	0.000	0.037	0.366	0.317	0.122	0.000	0.000	0.000	0.683	0.963
100	0.000	0.037	0.366	0.305	0.122	0.000	0.000	0.000	0.671	0.963

b) ε = 0.5 η= 0

Temporal Tolerance (ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.293	0.537	0.963	0.500	0.110	0.293	0.488	0.585	0.963	1.000
10	0.000	0.061	0.476	0.378	0.110	0.000	0.012	0.098	0.841	1.000
20	0.000	0.049	0.415	0.305	0.098	0.000	0.000	0.037	0.768	0.988
30	0.000	0.049	0.415	0.305	0.098	0.000	0.000	0.037	0.768	0.988
40	0.000	0.049	0.415	0.305	0.098	0.000	0.000	0.037	0.768	0.988
50	0.000	0.049	0.415	0.305	0.098	0.000	0.000	0.037	0.768	0.988
60	0.000	0.049	0.415	0.305	0.098	0.000	0.000	0.037	0.768	0.988
70	0.000	0.049	0.415	0.305	0.098	0.000	0.000	0.037	0.768	0.988
80	0.000	0.049	0.415	0.293	0.098	0.000	0.000	0.037	0.756	0.988
90	0.000	0.049	0.415	0.293	0.098	0.000	0.000	0.037	0.756	0.988
100	0.000	0.049	0.415	0.293	0.098	0.000	0.000	0.037	0.756	0.988

Table 5.7: Temporal tolerance of the ECG-105 with MA

a) Noisy ECG record: 105 (No. of R-peaks = 82)

Temporal Tolerance (ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.012	0.049	0.085	0.390	0.463	0.012	0.049	0.085	0.268	0.939
10	0.000	0.000	0.000	0.122	0.354	0.000	0.000	0.000	0.000	0.829
20	0.000	0.000	0.000	0.122	0.317	0.000	0.000	0.000	0.000	0.793
30	0.000	0.000	0.000	0.122	0.305	0.000	0.000	0.000	0.000	0.780
40	0.000	0.000	0.000	0.122	0.305	0.000	0.000	0.000	0.000	0.780
50	0.000	0.000	0.000	0.122	0.305	0.000	0.000	0.000	0.000	0.780
60	0.000	0.000	0.000	0.122	0.305	0.000	0.000	0.000	0.000	0.780
70	0.000	0.000	0.000	0.122	0.293	0.000	0.000	0.000	0.000	0.768
80	0.000	0.000	0.000	0.122	0.280	0.000	0.000	0.000	0.000	0.756
90	0.000	0.000	0.000	0.122	0.256	0.000	0.000	0.000	0.000	0.732
100	0.000	0.000	0.000	0.122	0.256	0.000	0.000	0.000	0.000	0.732

b) ε= 0 η= 0.5

Temporal Tolerance (ms)	Insertion Error					Detection Error				
	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB	SNR = 12 dB	SNR = 6 dB	SNR = 0 dB	SNR = -6 dB	SNR = -12 dB
0	0.061	0.085	0.195	0.622	1.110	0.061	0.085	0.098	0.207	0.524
10	0.000	0.000	0.098	0.415	0.683	0.000	0.000	0.000	0.000	0.098

20	0.000	0.000	0.098	0.415	0.646	0.000	0.000	0.000	0.000	0.061
30	0.000	0.000	0.098	0.415	0.634	0.000	0.000	0.000	0.000	0.049
40	0.000	0.000	0.098	0.415	0.622	0.000	0.000	0.000	0.000	0.037
50	0.000	0.000	0.098	0.415	0.622	0.000	0.000	0.000	0.000	0.037
60	0.000	0.000	0.098	0.415	0.622	0.000	0.000	0.000	0.000	0.037
70	0.000	0.000	0.098	0.415	0.610	0.000	0.000	0.000	0.000	0.024
80	0.000	0.000	0.098	0.415	0.610	0.000	0.000	0.000	0.000	0.024
90	0.000	0.000	0.098	0.415	0.585	0.000	0.000	0.000	0.000	0.000
100	0.000	0.000	0.098	0.415	0.585	0.000	0.000	0.000	0.000	0.000

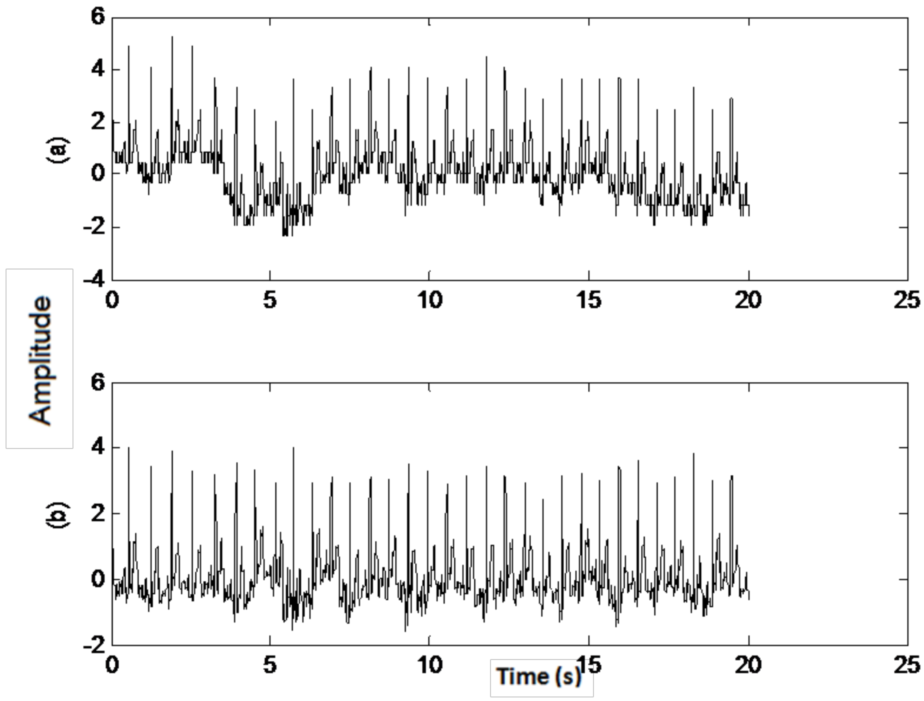

Figure 5.1: Suppression of artifact in ECG recorder using Holter monitor from 50 s to 70 s in walking condition (a) Input normalized ECG signal (b) processed output ECG with ε =0.1

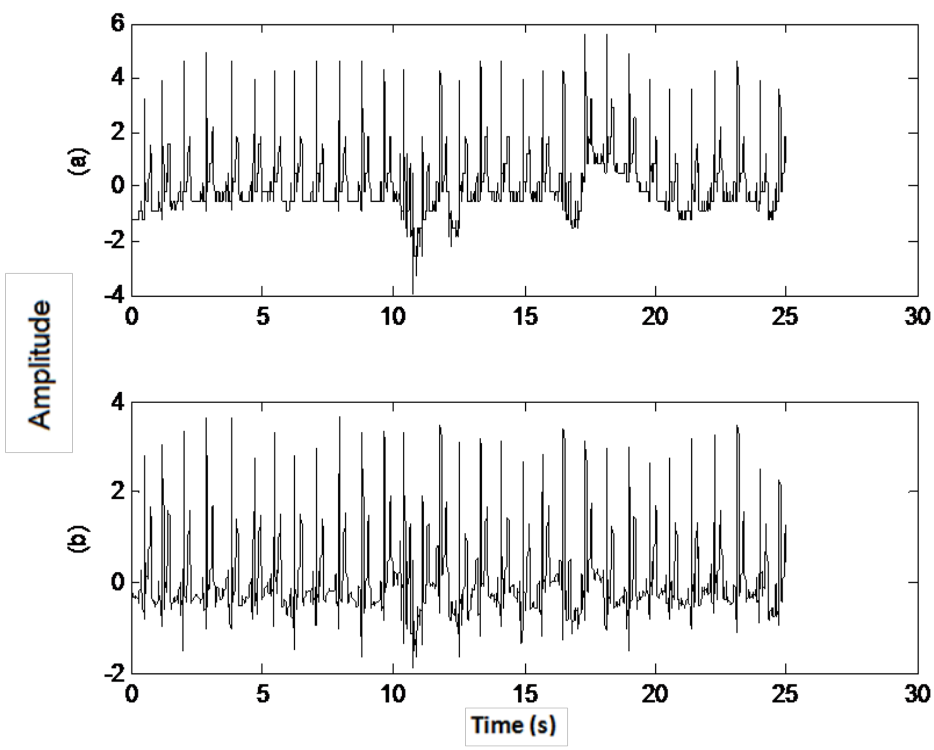

Figure 5.2: Suppression of artifact in ECG recorder using Holter monitor from 5 s to 30 s while getting seated (a) Input normalized ECG signal (b) processed output ECG with ε =0.05 and η =0.1

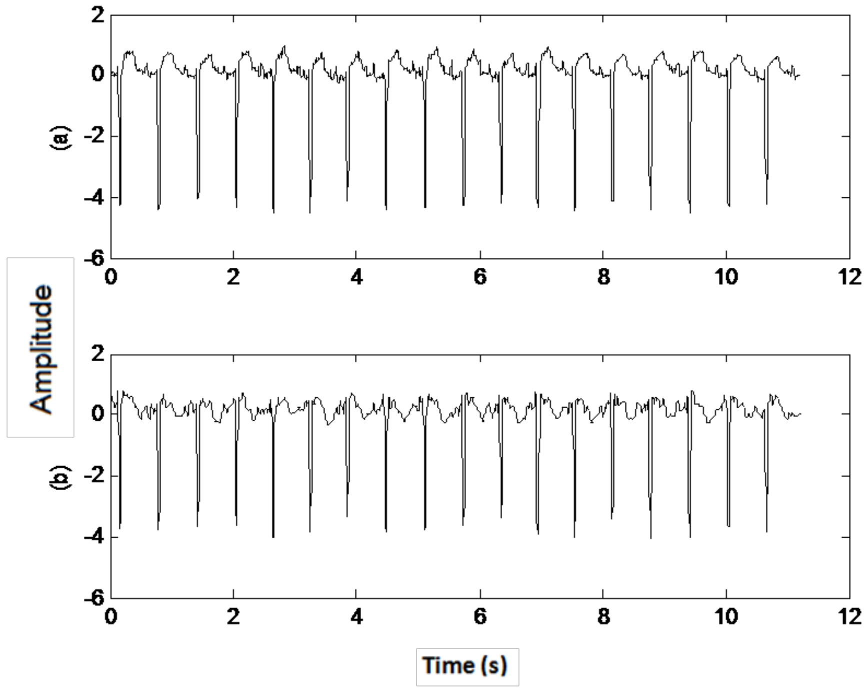

Figure 5.3: Suppression of artifact in ECG record-04908 of "afdb" database (a) Input normalized ECG signal (b) processed output ECG with ε =0.1 and η =0.1

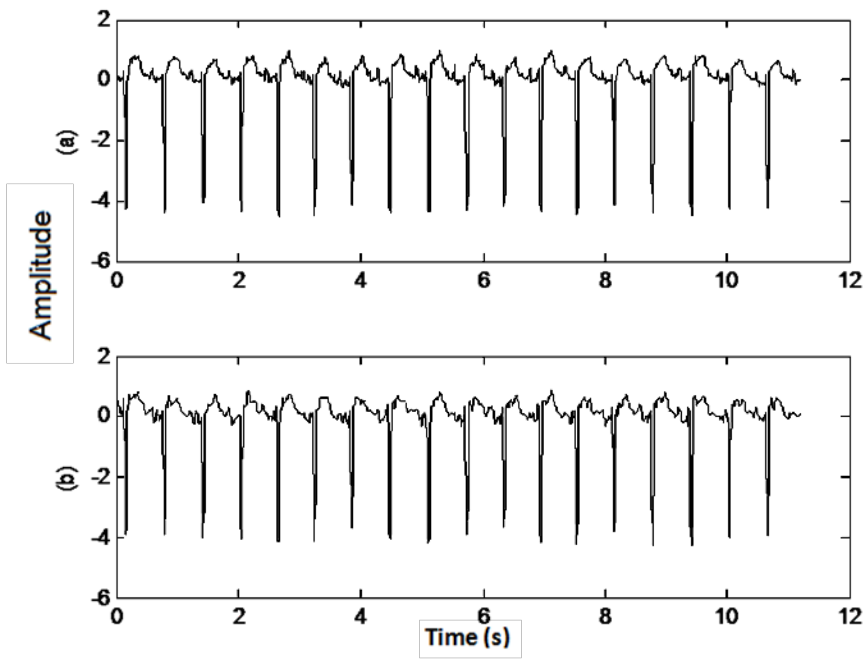

Figure 5.4: Suppression of artifact in ECG record-04908 of "afdb" database (a) Input normalized ECG signal (b) processed output ECG with ε =0.1 and η =0.

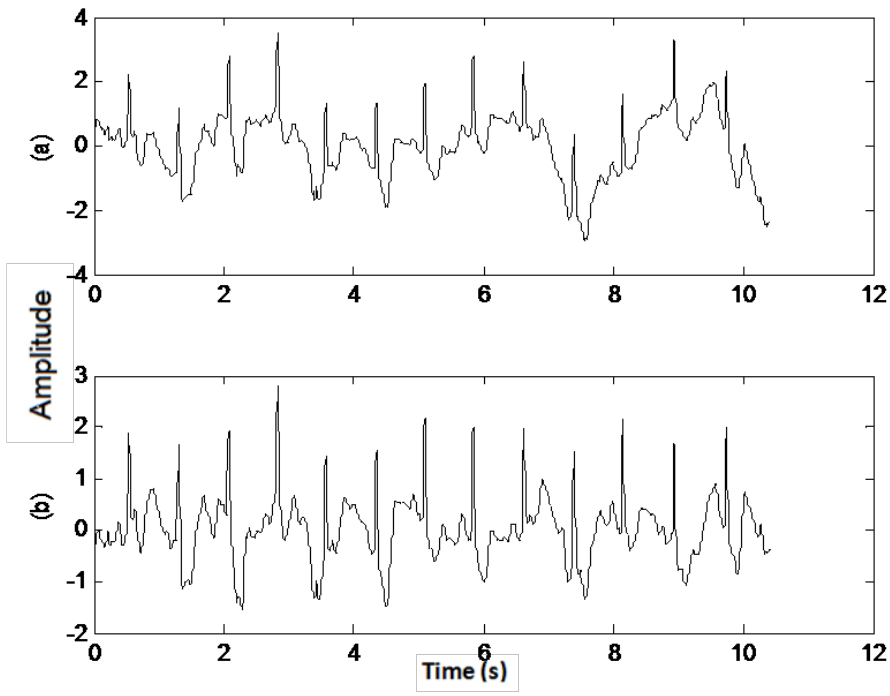

Figure 5.5: Suppression of artifact in first 10.4 s of ECG record-30 of "sddb" database (a) Input normalized ECG signal (b) processed output ECG with ε = 0.2

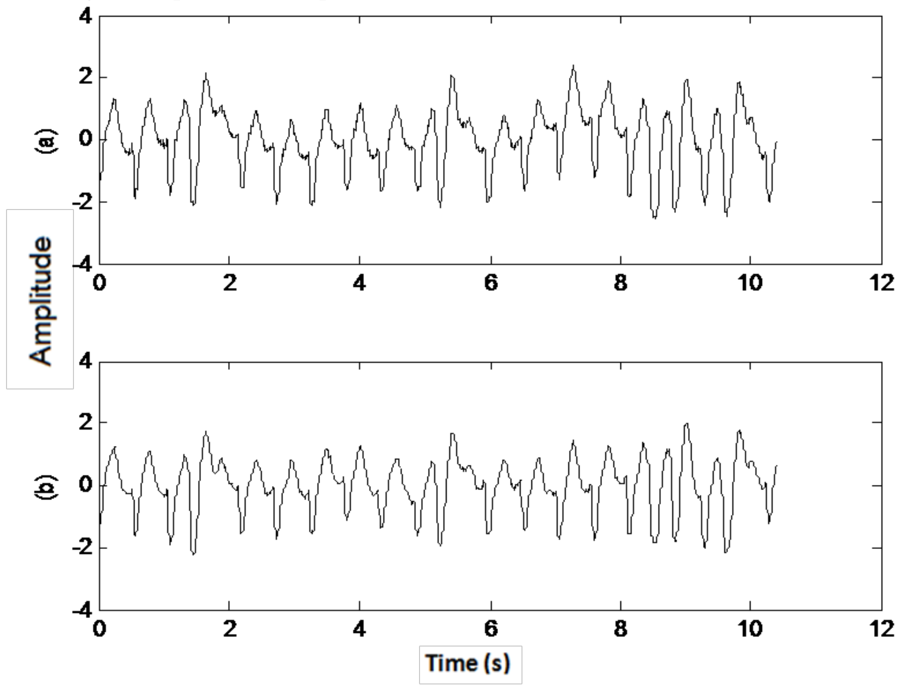

Figure 5.6: Suppression of artifact in first 10.4 s of ECG record-418 of "vfdb" database (a) Input normalized ECG signal (b) processed output ECG with ε =0.2

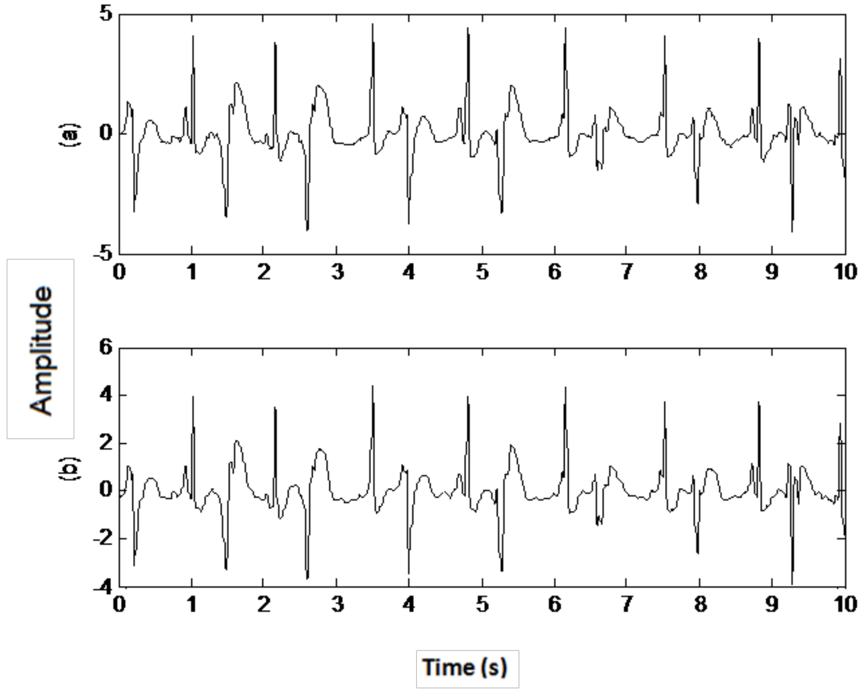

Figure 5.7: Suppression of artifact in first 10 s of ECG record-cu05 of "cudb" database (a) Input normalized ECG signal (b) processed output ECG with ε =0.1 and η =0

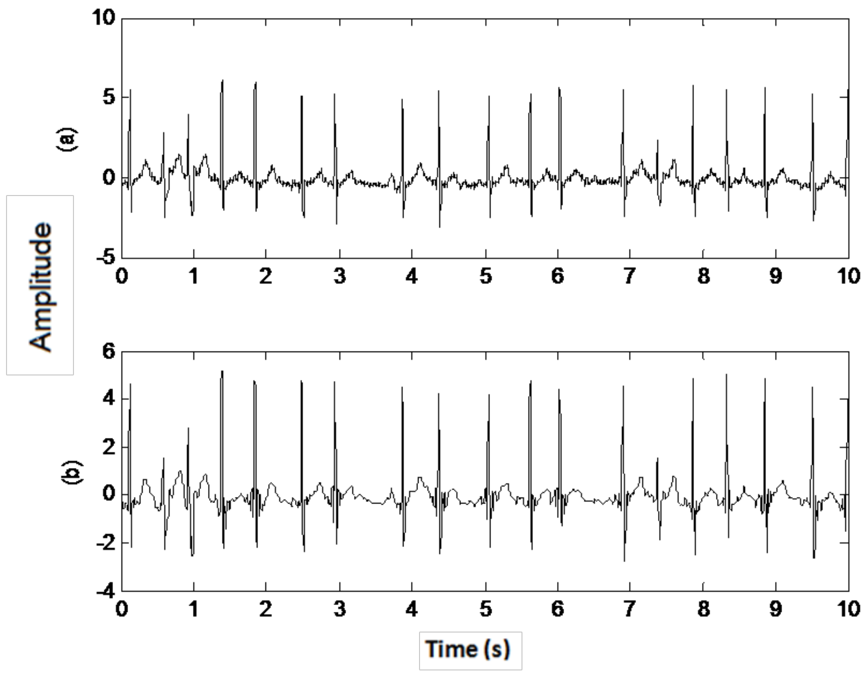

Figure 5.8: Suppression of artifact in first 10 s of ECG record-4 of "afdb" database (a) Input normalized ECG signal (b) processed output ECG with ε =0.1 and η =0.1

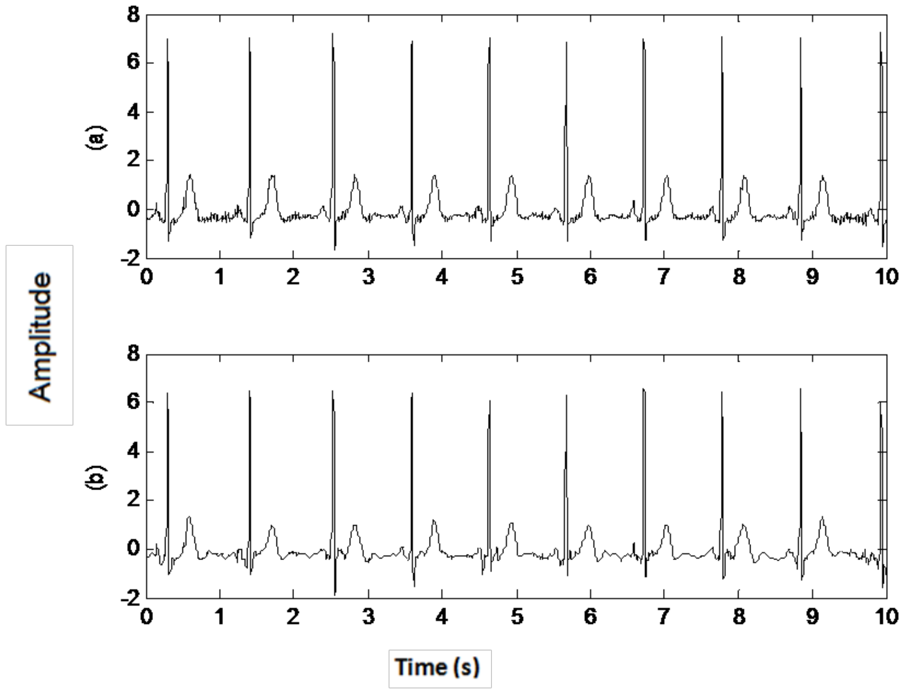

Figure 5.9: Suppression of artifact in first 10 s of ECG record-301 of "stdb" database (a) Input normalized ECG signal (b) processed output ECG with ε = 0.1.

Chapter 6
SUMMARY AND CONCLUSION

6.1 Summary

Ambulatory ECG recording are often corrupted by BW, EMG noise, and MA. These artifacts make it difficult to measure the duration and amplitude of P wave, time interval between characteristic points, dip or elevation of ST segments from isoelectric point. For suppression of these artifacts, investigations on a wavelet-based denoising technique was been carried out.

Objective evaluation of denoising techniques is generally carried out by applying them on ECG with simulated noise, obtained by adding noise-free ECG and ECG-free noise, and calculating several performance indices by using the noise-free ECG as the reference. SNR improvement (SNR_{impr}) and correlation coefficient (Corr. coeff.) are the most commonly used performance indices. They are based on sample-by-sample comparison of the processed output and the reference waveform. We have extended the use of correlation coefficient to decompose the error in the output with respect to the noise-free reference to get the estimates of signal attenuation (β), noise attenuation (γ), and distortion coefficient (κ). Measures based on comparison of signal statistics include improvement indices based on RMS and max-min values: II(RMS), RR(MM). We have proposed and investigated the use of these indices with reference to skewness, kurtosis, and cumulative distribution function: II(Skew.), II(Kurt.), E(CDF)_in, and E(CDF)_out. These selected and proposed performance indices can be used for providing comprehensive evaluation of denoising techniques and to get an insight for further improvement. We have also developed an automated method for calculating insertion and detection errors in R-peak detection as a function of temporal tolerance. These evaluation method can be particularly useful in the application of the denoising technique for arrhythmia detection.

A wavelet-based denoising technique, developed earlier in our lab, as reported in [16]-[18], using discrete Meyer wavelet (dmey), smooth thresholding and smooth limiting of wavelet coefficients, and thresholds determined from the signal statistics and externally supplied control parameters has been investigated. After empirical investigations, minor changes in the threshold determination and thresholding process were introduced. Its application on ECG with simulated noise and on ambulatory ECG was used to investigate the effect of the denoising control parameters. was suppressed by level-dependent thresholding technique.

6.2 Conclusions

A combination of control parameters which gives large SNR_{impr} at low input SNR without a very large signal degradation at high input SNR may be considered as an optimal combination. Based on this consideration, the optimal combinations for denoising of different artifacts using the reported denoising technique and SNR_{impr} at input SNR of −12 dB were found to be as the following:

BW: $\varepsilon = 0$ and $\eta = 0$, $SNR_{impr} = 17.6$ dB.

EMG noise: $\varepsilon = 0.5$ and $\eta = 0$, $SNR_{impr} = 5.9$ dB.

MA: $\varepsilon = 0$ and $\eta = 0.5$, $SNR_{impr} = 9.3$ dB

Although the technique results in large SNR_{impr} for BW, further examination of the denoising using error decomposition showed scope for further improvement by applying level-dependent thresholding on $D_1(n)$ and $A_8(n)$, rather than setting them to zero. Examination of the results for denoising of EMG noise and MA showed that selection of the combination of denoising control parameters should be made based on an assessment of the level and type of artifacts: a higher ε for high level of EMG noise and higher η for high level of MA.

Results of insertion and detection errors showed that the Pan-Tompkins algorithm [47] has a very low sensitivity to BW, and therefore it can be used for arrhythmia detection in the presence of BW. It is highly susceptible to EMG noise and the denoising technique was not able to provide any significant improvement. The denoising technique was found to be useful in improving R-peak detection in the presence of MA.

6.3 Suggestions for further work

Evaluation of the denoising technique needs to be carried out for signals corrupted with a mix of BW, EMG noise, and MA. Investigations on robust methods for determining the thresholds from the signal statistics and with appropriate temporal variability may help in improving usefulness of the technique. Devising the technique without the need for externally provided denoising control parameters can extend its usefulness for use in Holter recorders.

Appendix A

DESCRIPTION OF FUNCTIONS USED IN SIGNAL PROCESSING AND CALCULATION OF PERFORMANCE INDICES

1. Functions used for signal processing

emg_denoise.mat, ma_denoise.mat, mixSNR.mat, normalized_zero_mean.mat are the functions used for signal processing.

2. Functions used for calculation of performance indices

allindices.mat, temporal_accuracy.mat, getSNR.mat are the functions used for calculation of perfromance indices.

3. Description of functions

3.1 allindices.mat

Calculates the performance indices used for evaluation of the denoising, from the arrays containing noise-free ECG, noisy ECG, and denoised ECG signals, and returns an array with the results.

result = allindices (ECG_original, ECG_noisy, ECG_denoised)

result = { ECG indices (RMS, maxmin, skewness, kurtosis, corr. coeff., E-CDF), noisy ECG indices (RMS, .., E-CDF), denoised ECG indices (RMS, .., E-CDF), SNR of ECG, SNR of noisy, SNR of denoised, alpha, beta, gamma, kappa)

Calculation are carried out using the following as signal, noisy signal, noise, and output

$\mathbf{s}$ = {ECG_original}, $\mathbf{x}$= {ECG_noisy}, $\alpha\mathbf{d} = \mathbf{x} - \mathbf{s}$, $\mathbf{y}$= {ECG_denoised}

and using the methods as described in Sections 3.3 - 3.6.

3.2 emg_denoise.mat

EMG denoising of ECG signal

denoised_ECG_signal = emg_denoise (noisy_ECG_signal, emgCtrl)

x = {noisy_ECG_signal}, **y** = {denoised_ECG_signal}, ε = emgCtrl

Algorithm used is as described in Section 4.2.1. From taking IDWT of processed ECG y is obtained.

3.3 getSNR.mat

Calculation of SNR (dB) of noisy signal with reference to noise-free reference.

SNR = getSNR (noisy_signal, reference_signal)

The first array is taken as the reference signal and the second array is taken as the noisy signal. The difference of the two is taken as the noise and used to calculate the SNR (dB).

x = {noisy_signal}, **s** = {reference_signal}

SNR $\mathbf{= 20\ log(\|s\| / \|x - s\|)}$

3.4 ma_denoise.mat

MA denoising of ECG signal

denoised_ECG_signal = ma_denoise (noisy_ECG_signal, maCtrl)

x = {noisy_ECG_signal}, **y** = {denoised_ECG_signal}, η = maCtrl

Algorithm used is as described in Section 4.2.2.

3.5 mixSNR.mat

Mixing signal and noise at specified SNR (dB).

noisy_output = mixSNR (signal, noise, SNR)

The first array is taken as signal and the second array is taken as noise. The RMS values of the two are used to calculate the scaling factor, corresponding to the specified SNR (dB), for multiplying the second array. The second array is multiplied with the scaling factor and added to the first array and the resulting array is output as the noisy signal. It implements the equation 3.9 and 3.10

x = {noisy_output}, **s** = {reference_signal}, d= {noise}, SNR= {SNR}

3.6 normalized_zero_mean.mat

Normalizing the signal and with zero mean

output_signal= normalized_zero_mean (input_signal)

Calculates the RMS value of signal and divides by it. Mean of the signal is calculated and subtracted from it.

y = {output_signal}, x = {input_signal}

$\mathbf{y}=(\mathbf{x}-\hat{\mathbf{x}})/\|\mathbf{x}\|$

3.7 temporal_tolerance.mat

Calculates the temporal tolerance, from the arrays containing noise-free ECG, noisy ECG, and denoised ECG signals, and returns four arrays.

detection_error_denoised, insertion_error_denoised, insertion_error_noisy, detection_error_noisy = SNR temporal accuracy (ECG_original, ECG_noisy, ECG_denoised)

detection_error_denoised = {corresponding values to temporal tolerance of 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100 ms}

insertion_error_denoised = {corresponding values to temporal tolerance of 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100 ms}

insertion_error_denoised = {corresponding values to temporal tolerance of 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100 ms}

insertion_error_denoised = {corresponding values to temporal tolerance of 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100 ms}

and the calculation is described in Section 3.6.4

Appendix B

WAVEFORMS RELATED TO INVESTIGATIONS ON DENOISING

Waveforms of ECG with noise and

20 s ECG segment from record-16483 of "nsrdb" database corrupted by BW and EMG noise at SNR values of 0 dB and − 12 dB, and its denoising with various combination of ε and η. The first 3 s segment is repeated in the subsequent figure. 20 s ECG segment from record-219 of "mitdb" database corrupted by BW, EMG noise at SNR values of 0 dB and − 12 dB, and its denoising with various combination of ε and η. 20 s ECG segment from record-219 of "mitdb" database was corrupted by two types of motion artifacts and its denoising with various combination of ε and η are shown below.

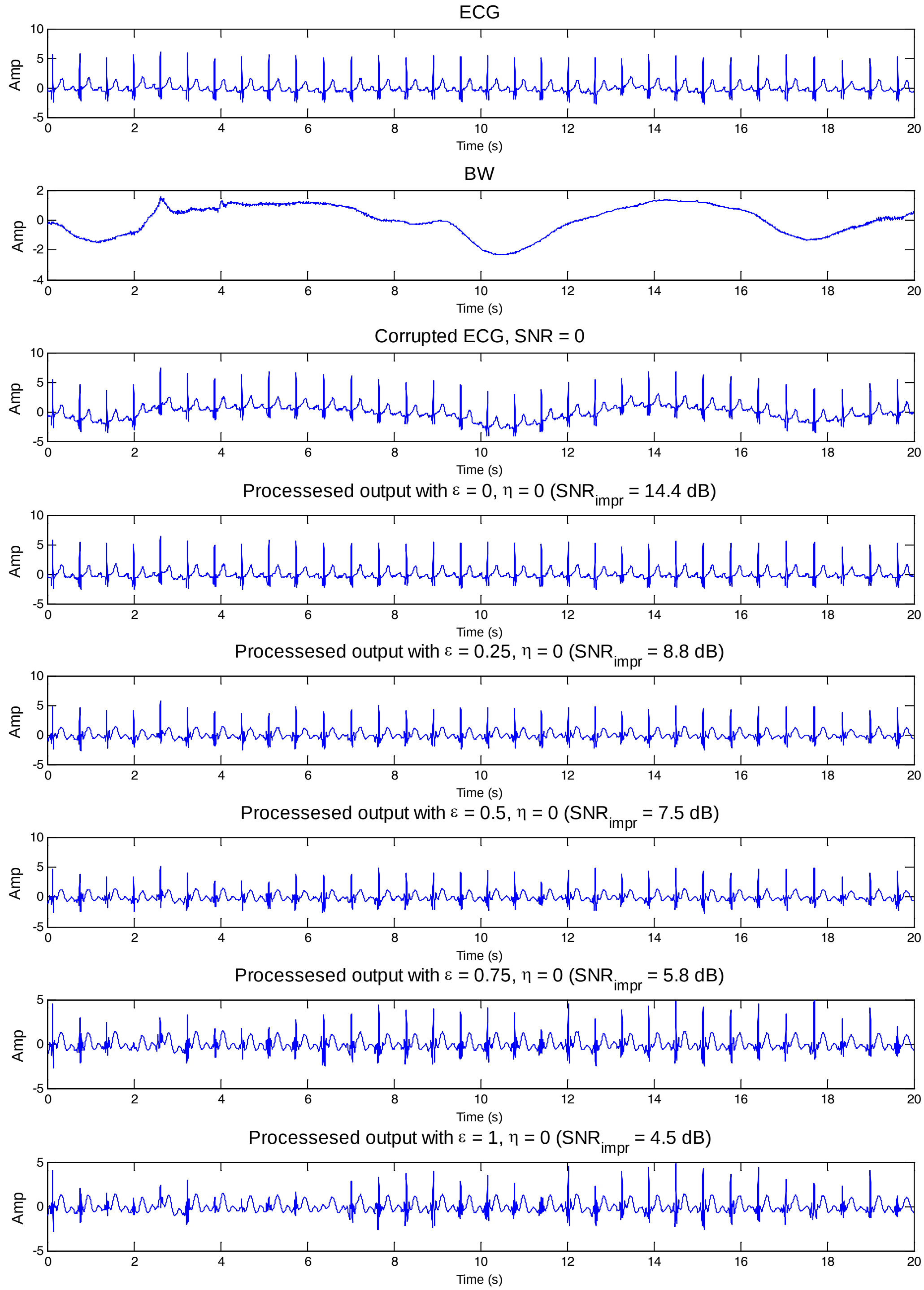

Figure B.1: Continued

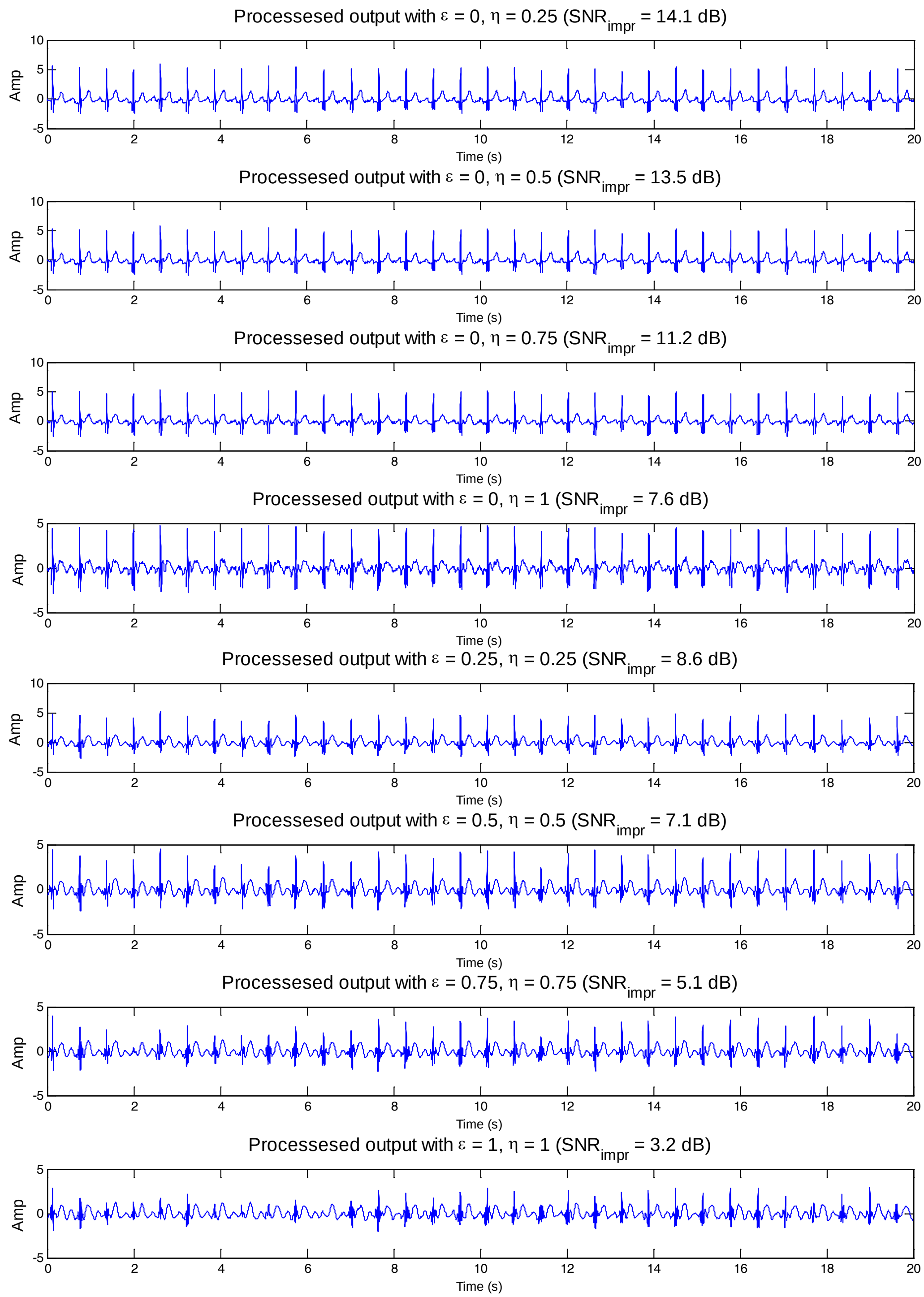

Figure B.1: 20-s ECG segment from record-16483 of "nsrdb" database corrupted by BW at SNR = 0 dB, and its denoising with various combination of ε and η.

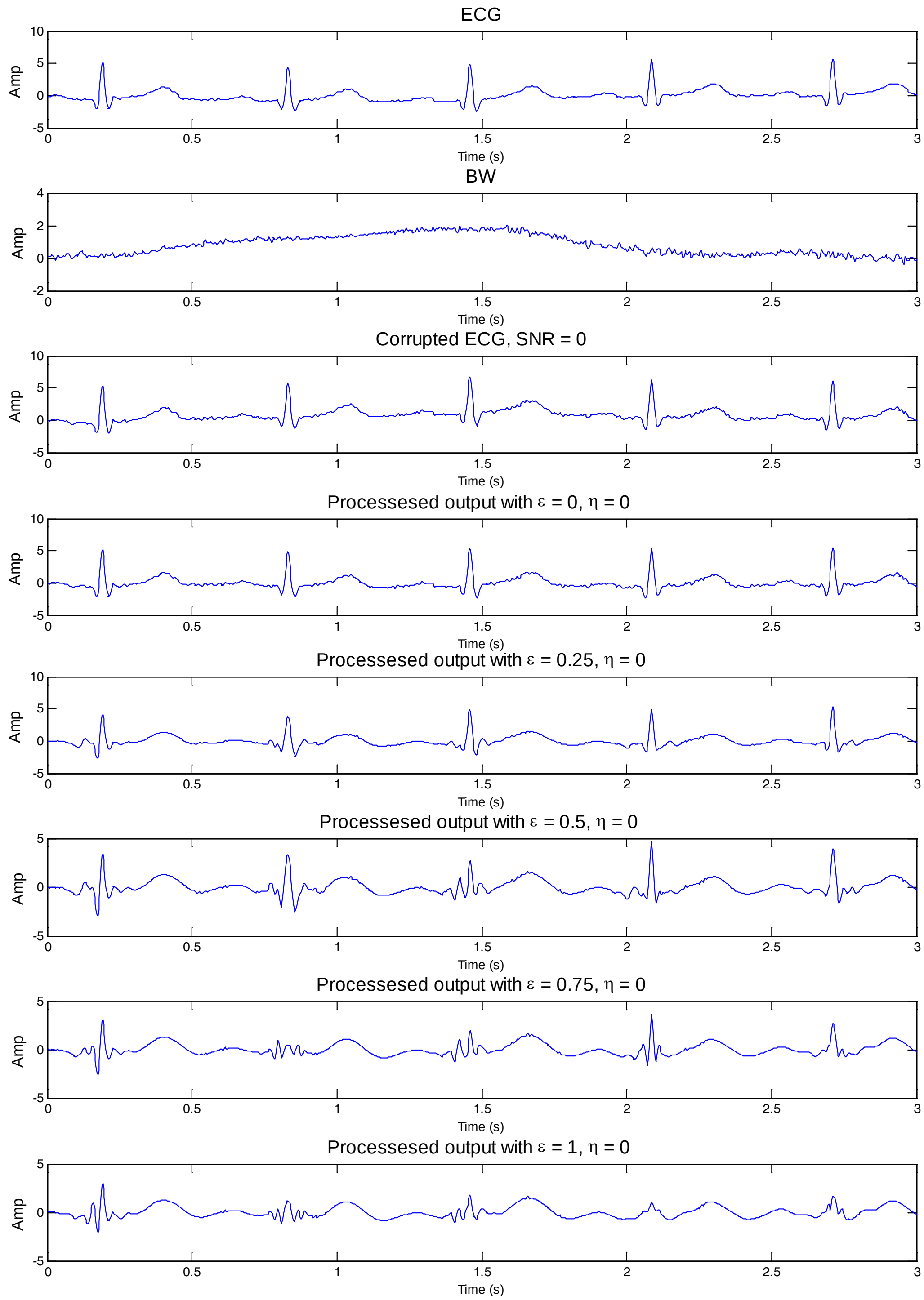

Figure B.2: Continued

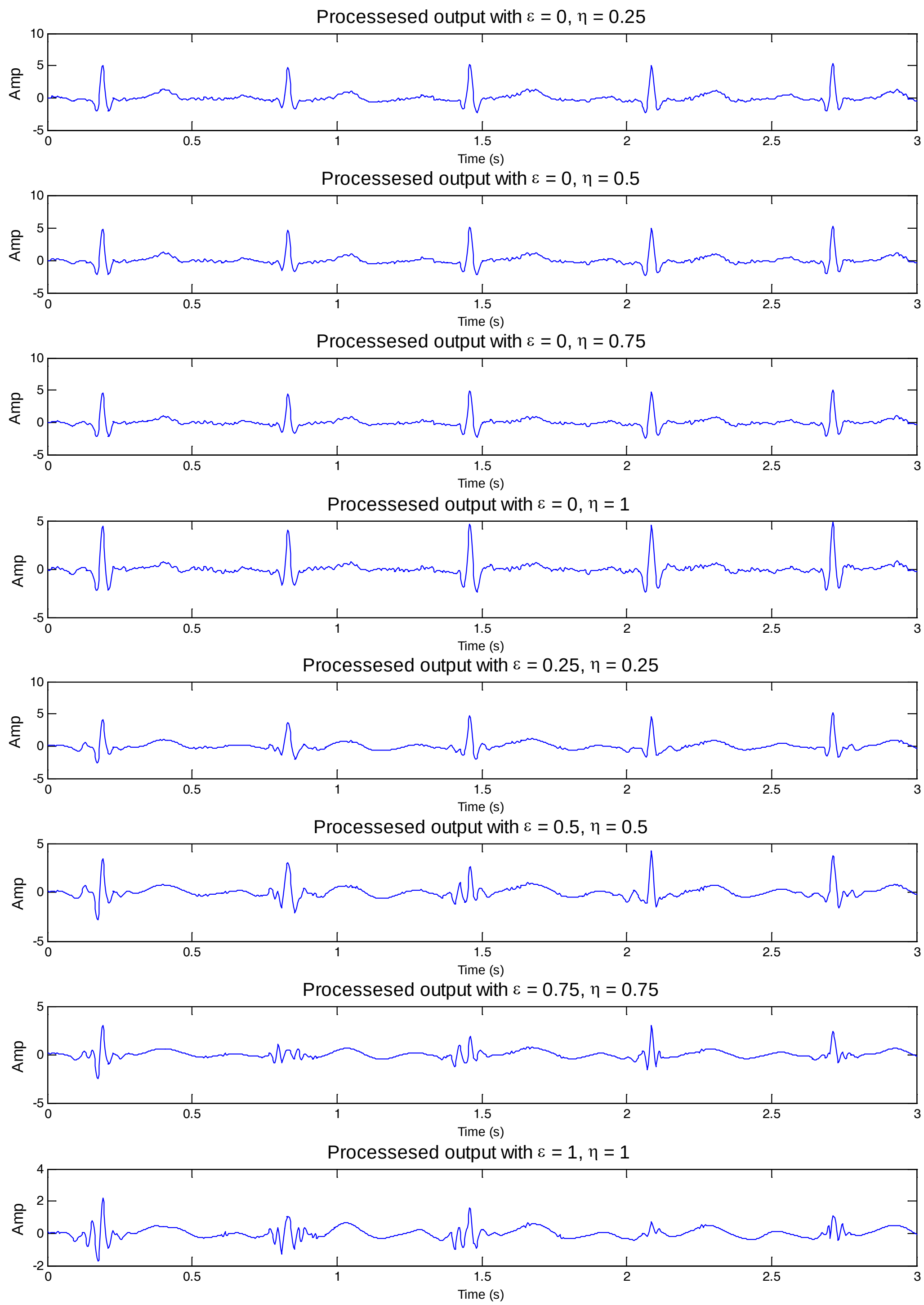

Figure B.2: 3-s ECG segment from record-16483 of "nsrdb" database corrupted by BW at SNR = 0 dB, and its denoising with various combination of ε and η.

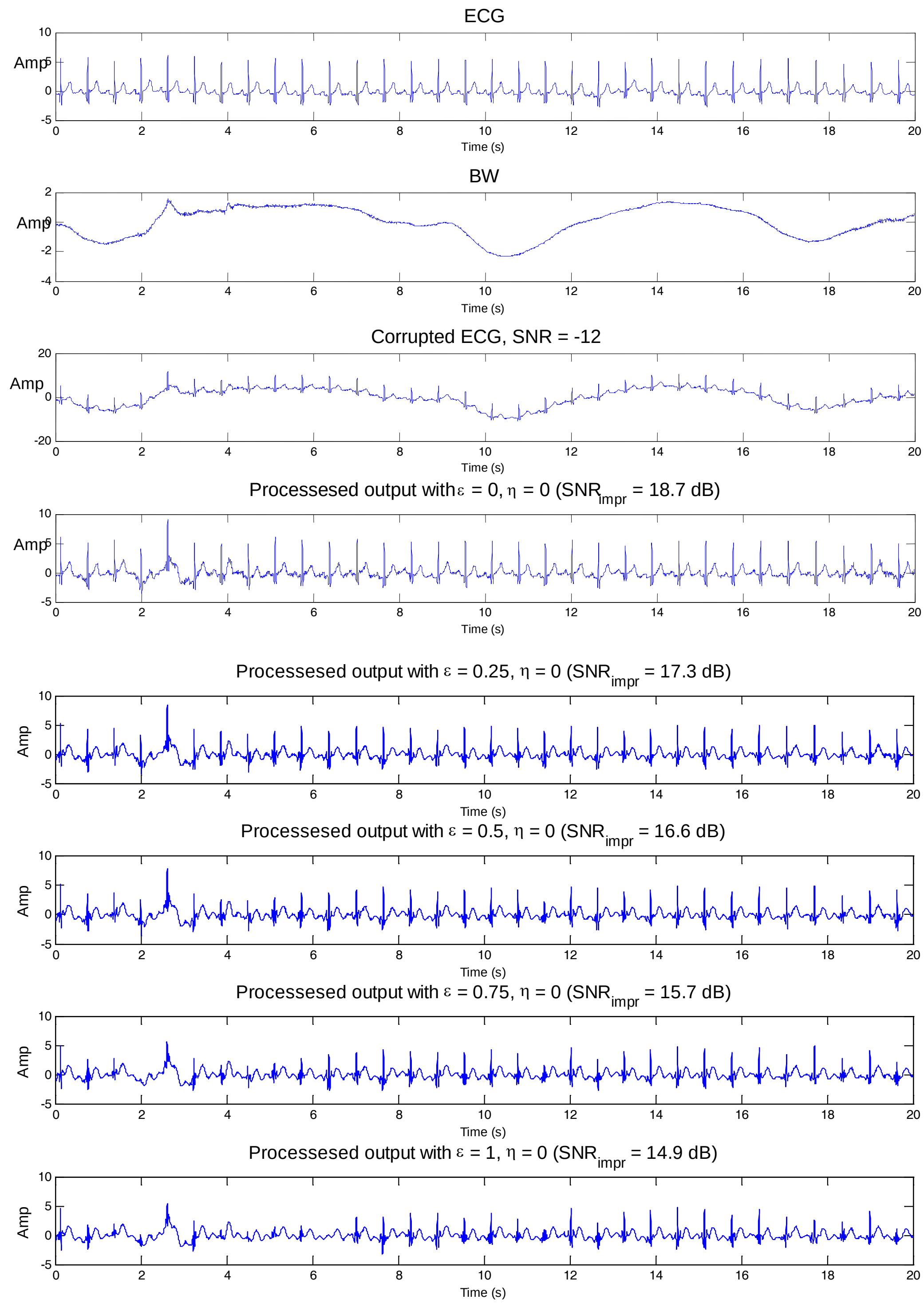

Figure B.3: Continued

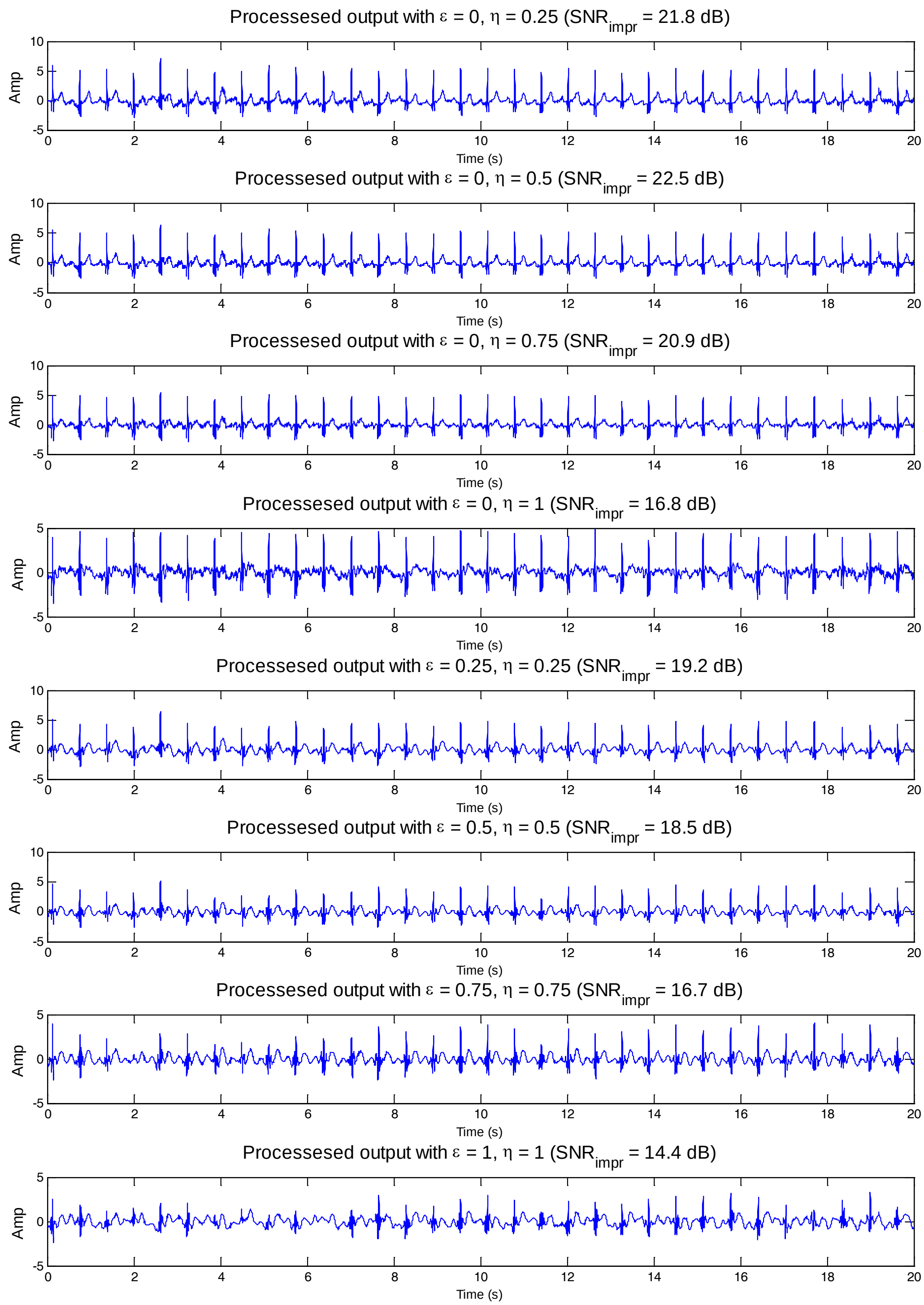

Figure B.3: 20-s ECG segment from record-16483 of "nsrdb" database corrupted by BW at SNR = − 12 dB, and its denoising with various combination of ε and η.

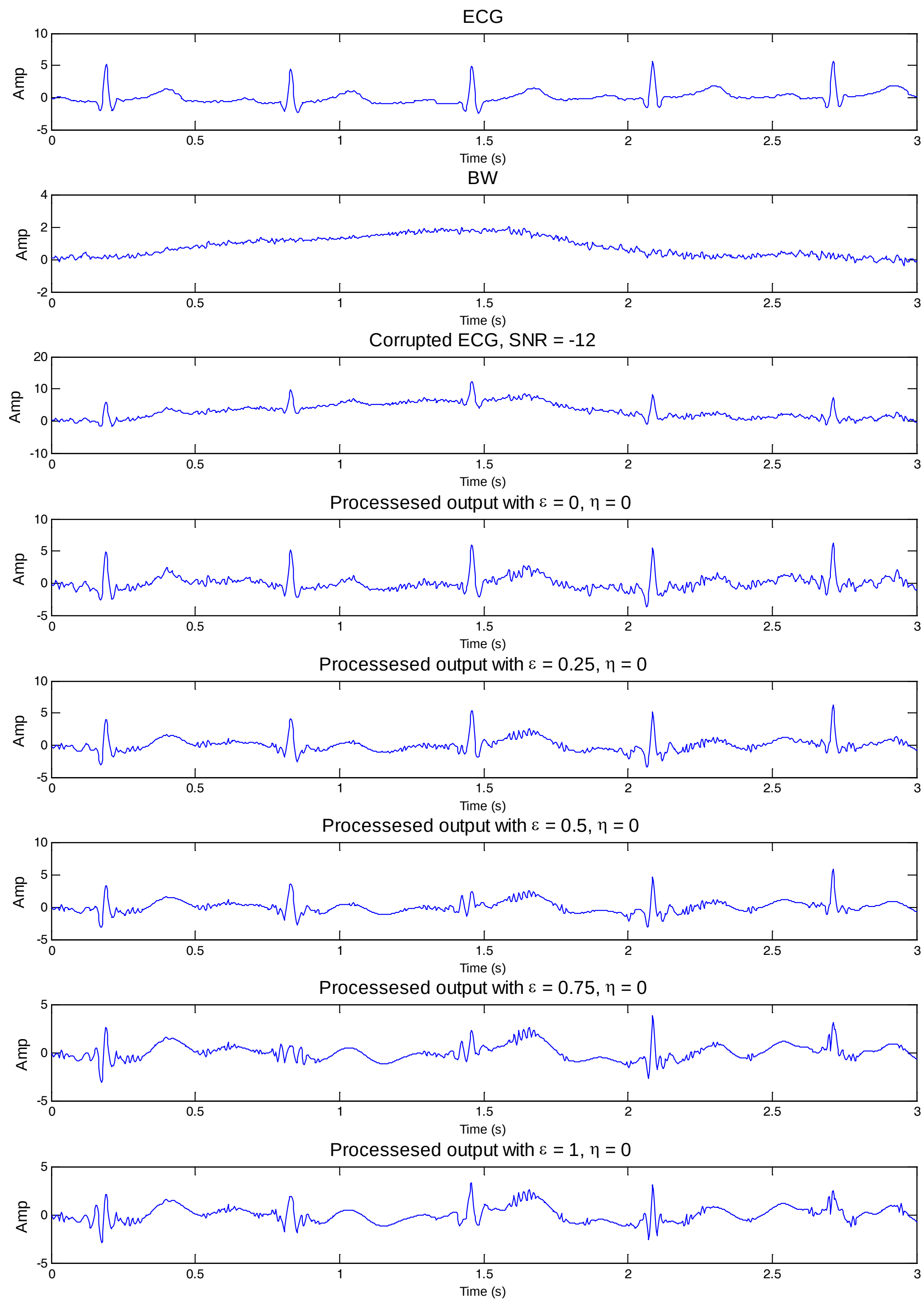

Figure B.4: Continued

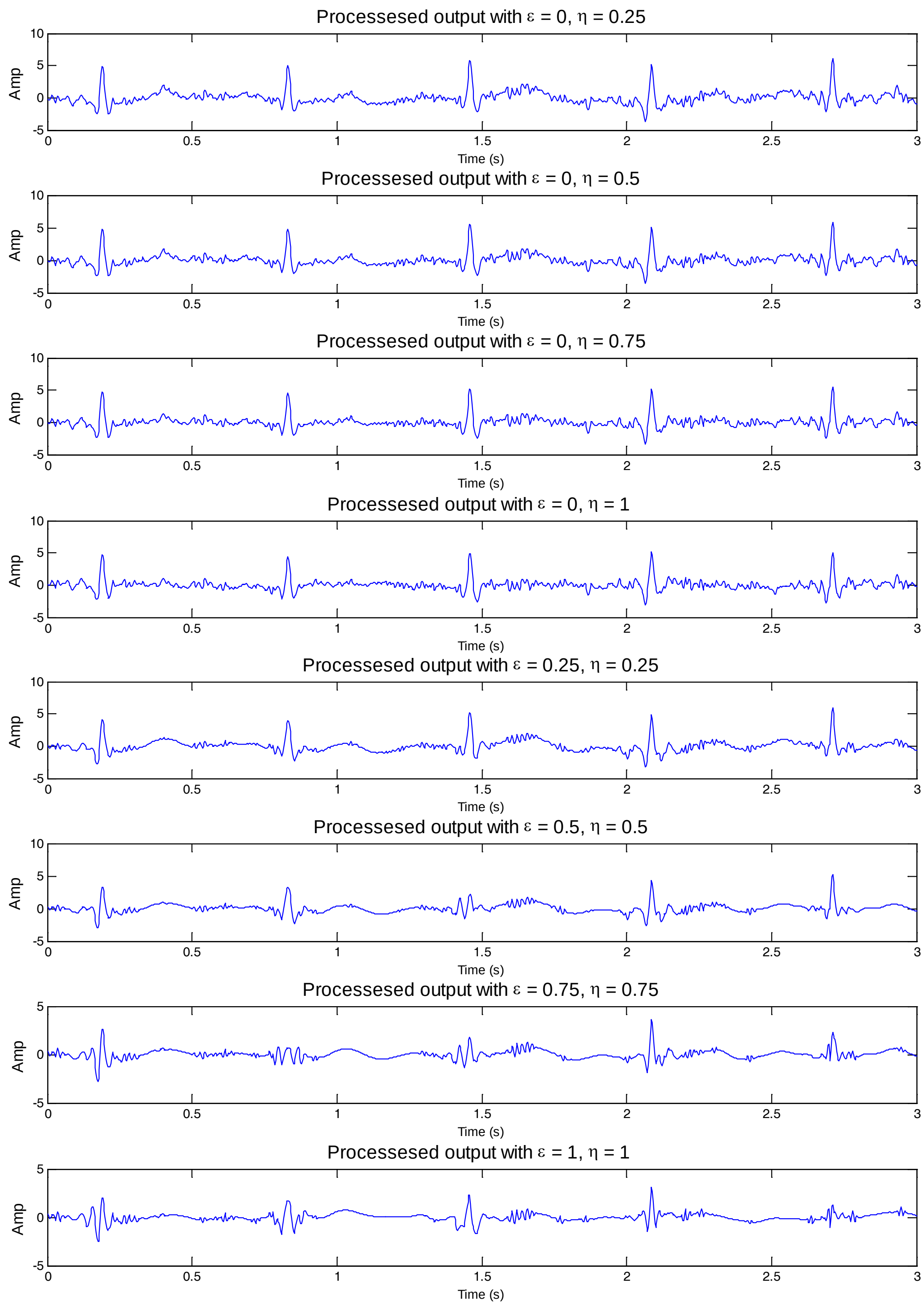

Figure B.4: 3-s ECG segment from record-16483 of "nsrdb" database corrupted by BW at SNR = − 12 dB, and its denoising with various combination of ε and η.

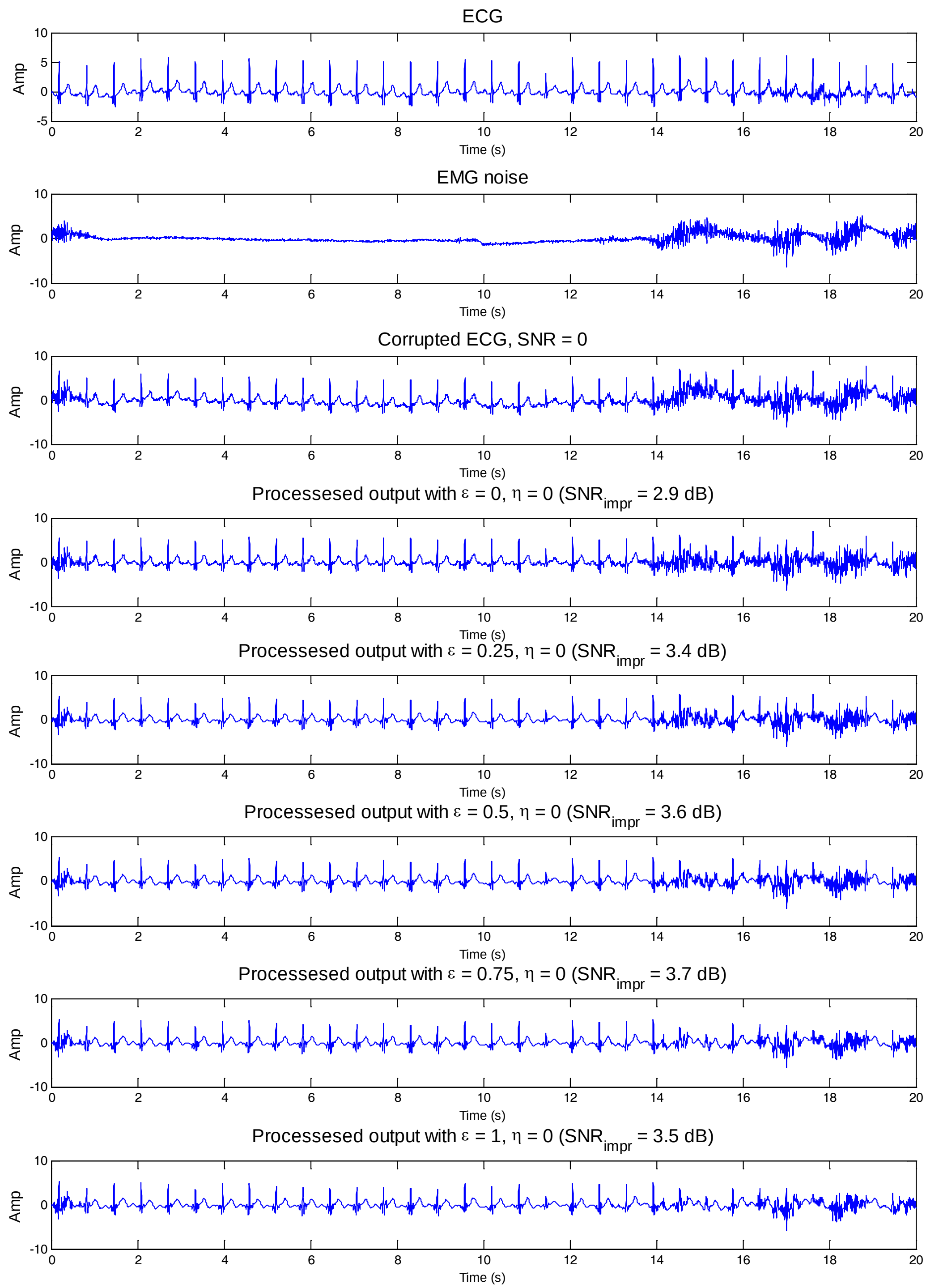

Figure B.5: Continued

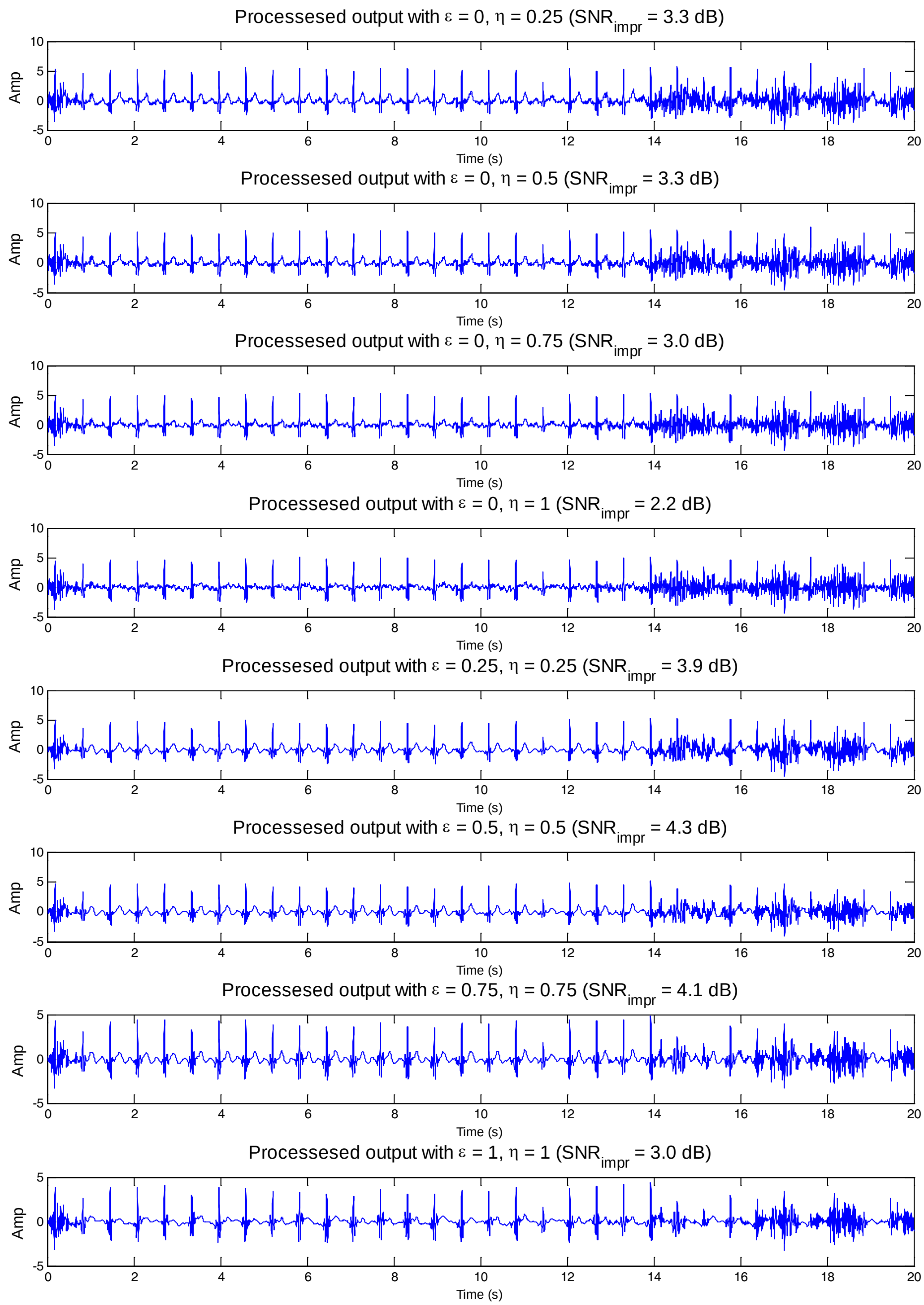

Figure B.5: 20-s ECG segment from record-16483 of "nsrdb" database corrupted by EMG noise at SNR = 0 dB, and its denoising with various combination of ε and η.

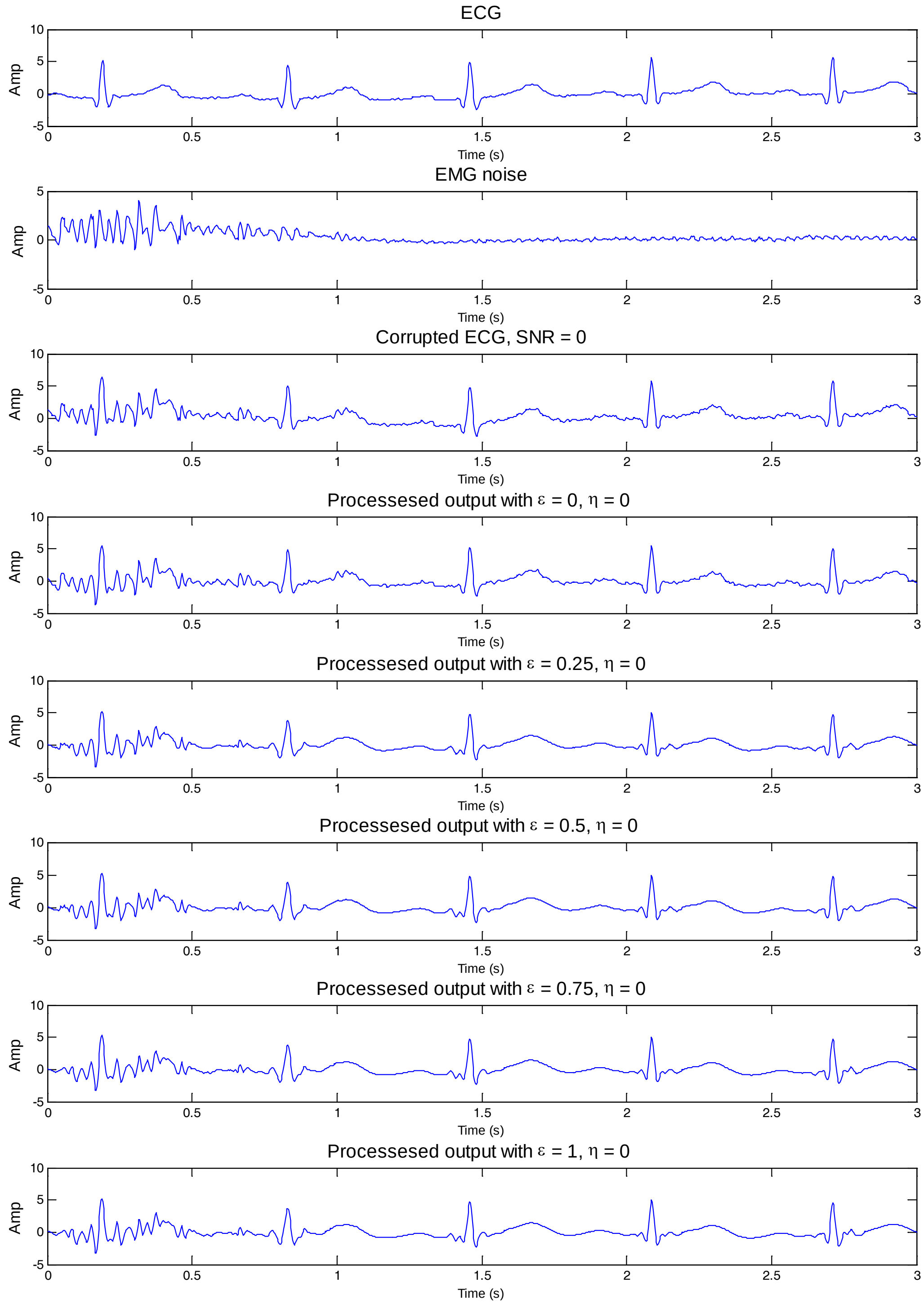

Figure B.6: Continued

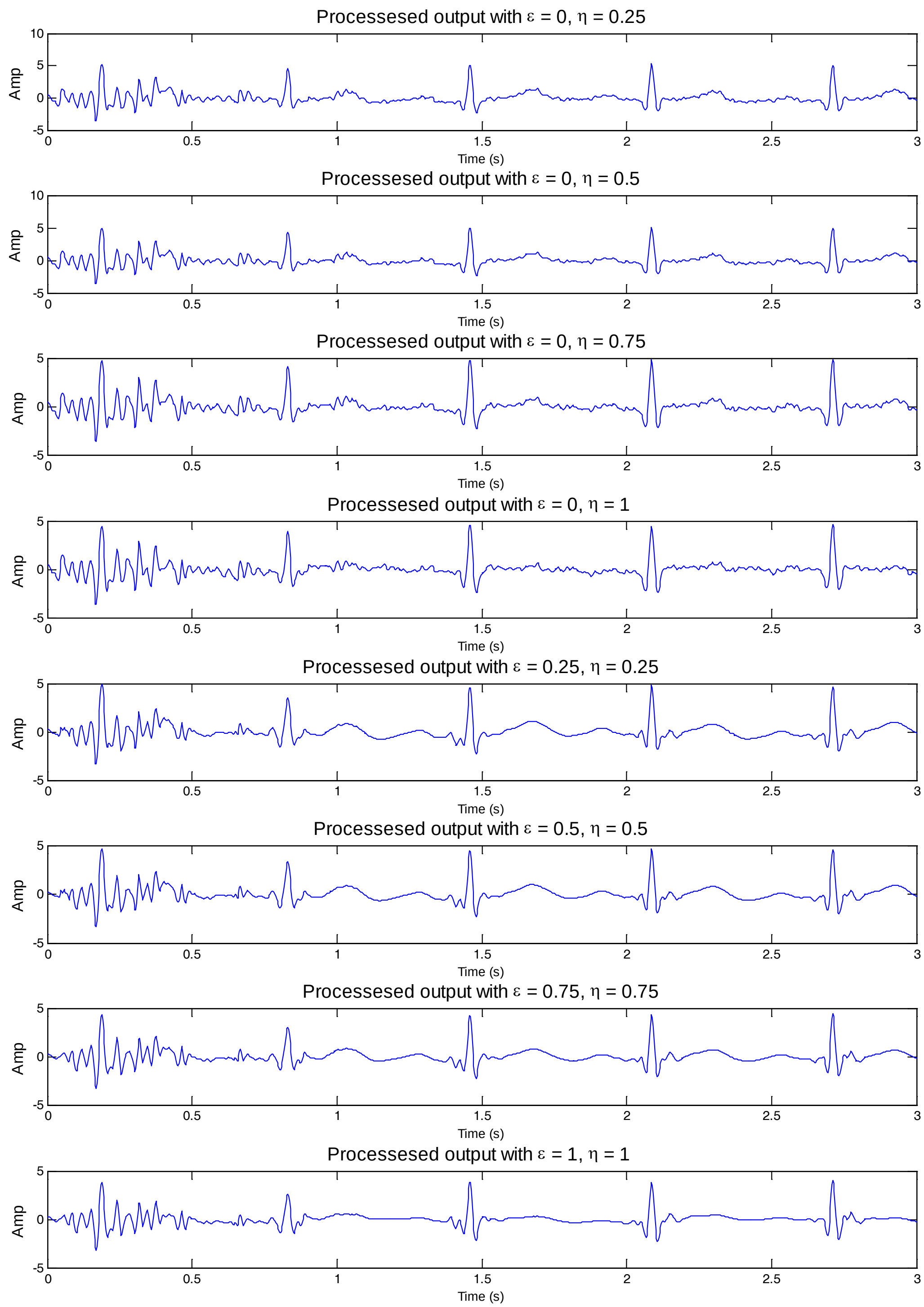

Figure B.6: 3-s ECG segment from record-16483 of "nsrdb" database corrupted by EMG noise at SNR = 0 dB, and its denoising with various combination of ε and η.

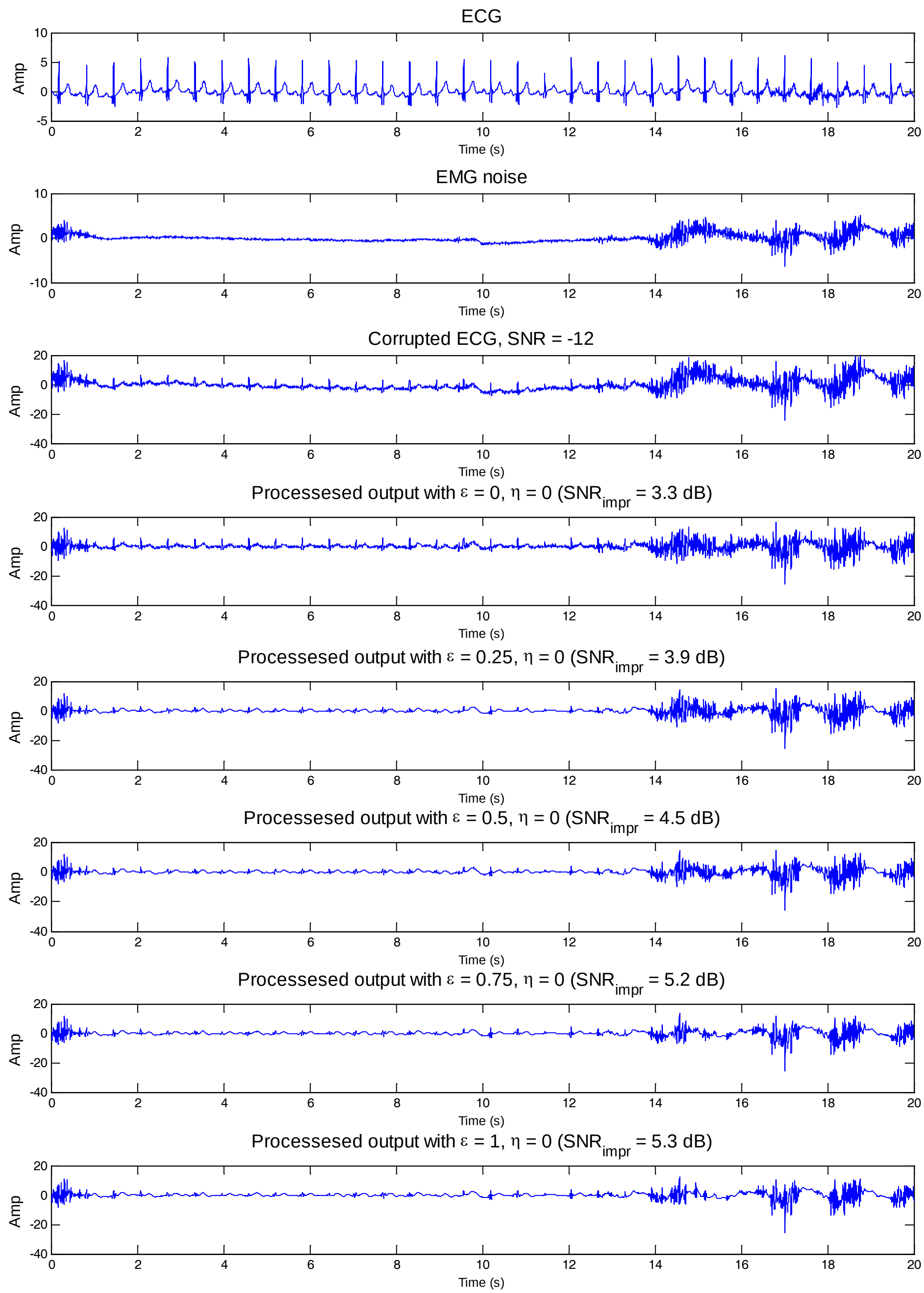

Figure B.7: Continued

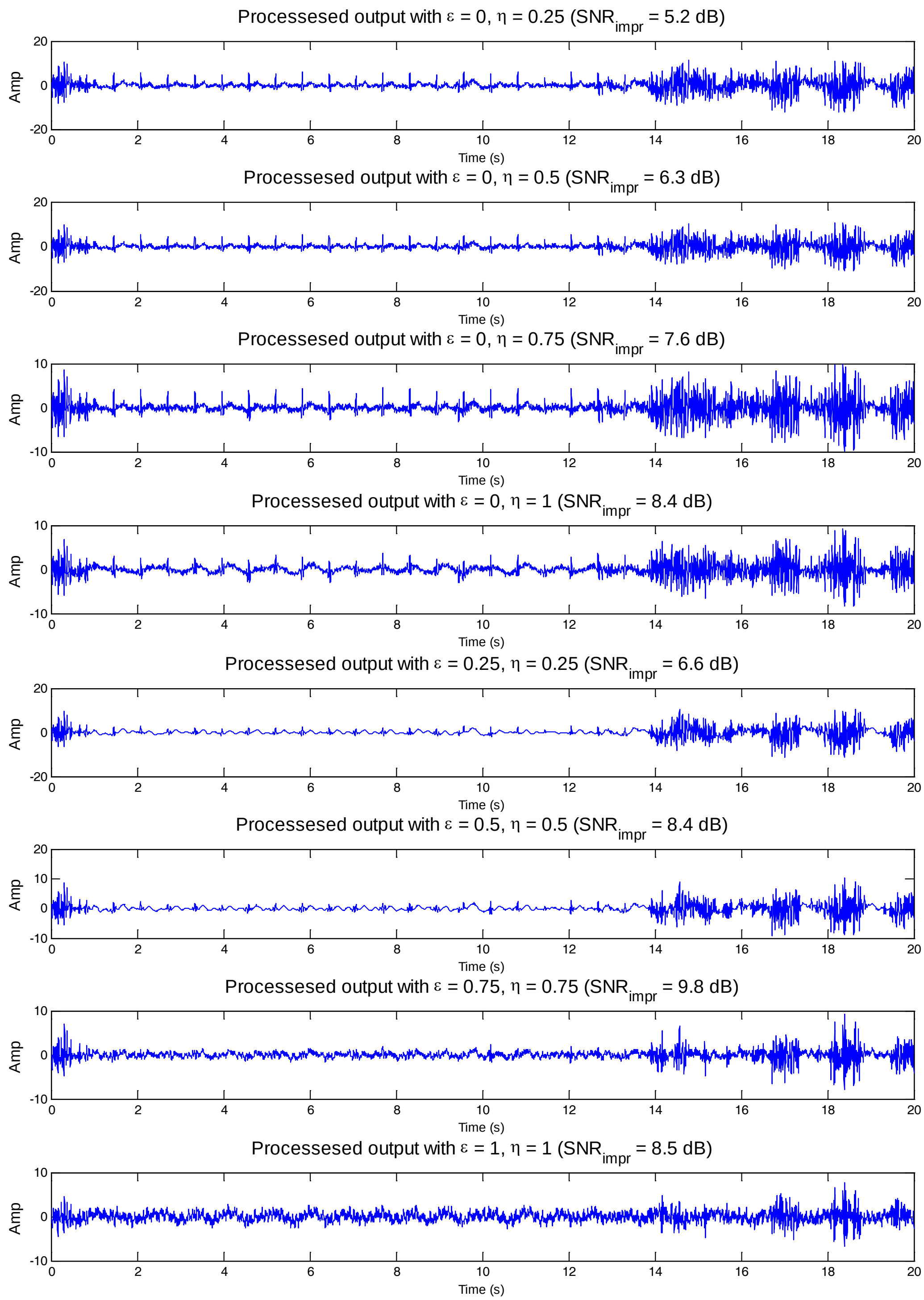

Figure B.7: 20-s ECG segment from record-16483 of "nsrdb" database corrupted by EMG noise at SNR = − 12 dB, and its denoising with various combination of ε and η.

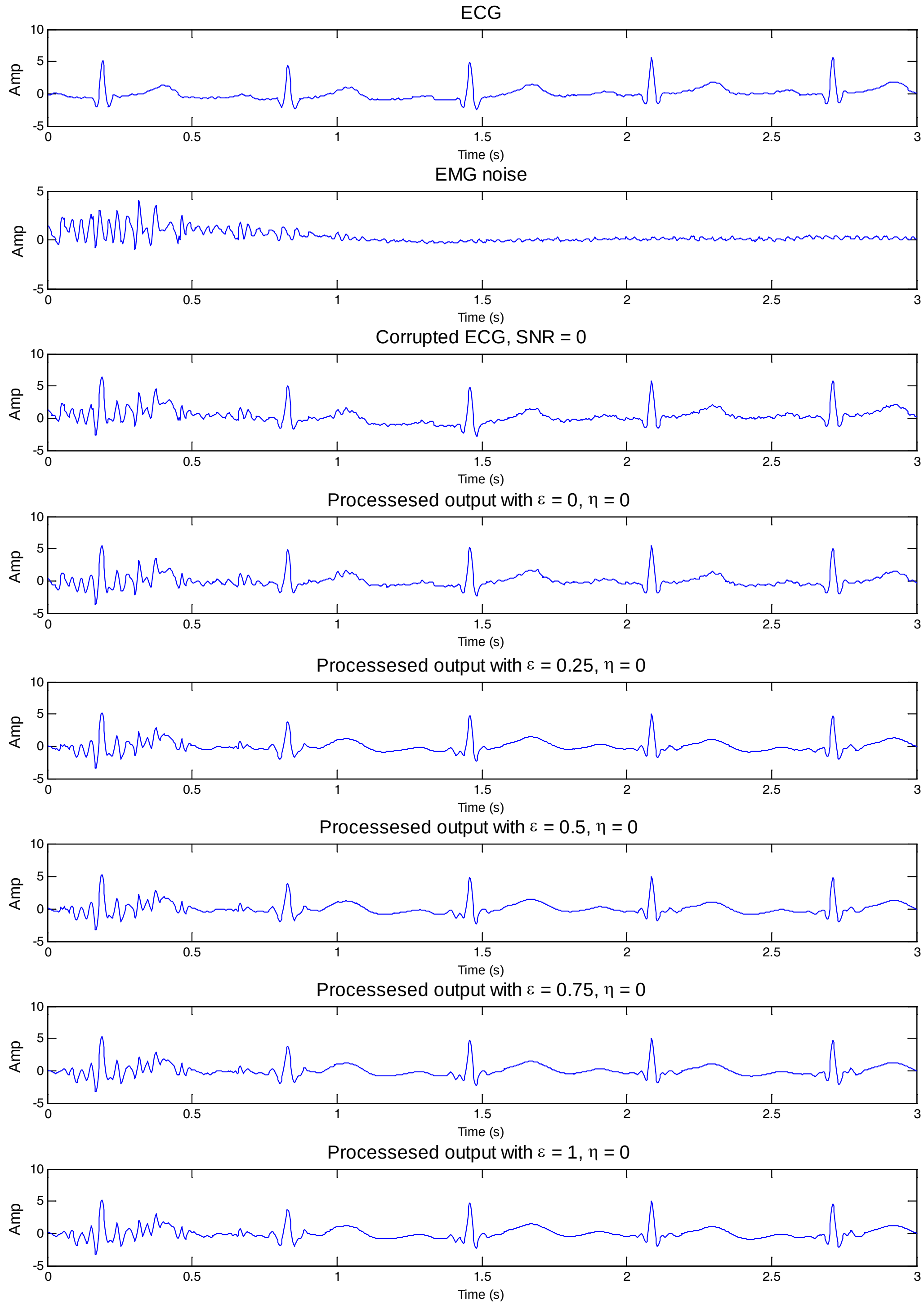

Figure B.8: Continued

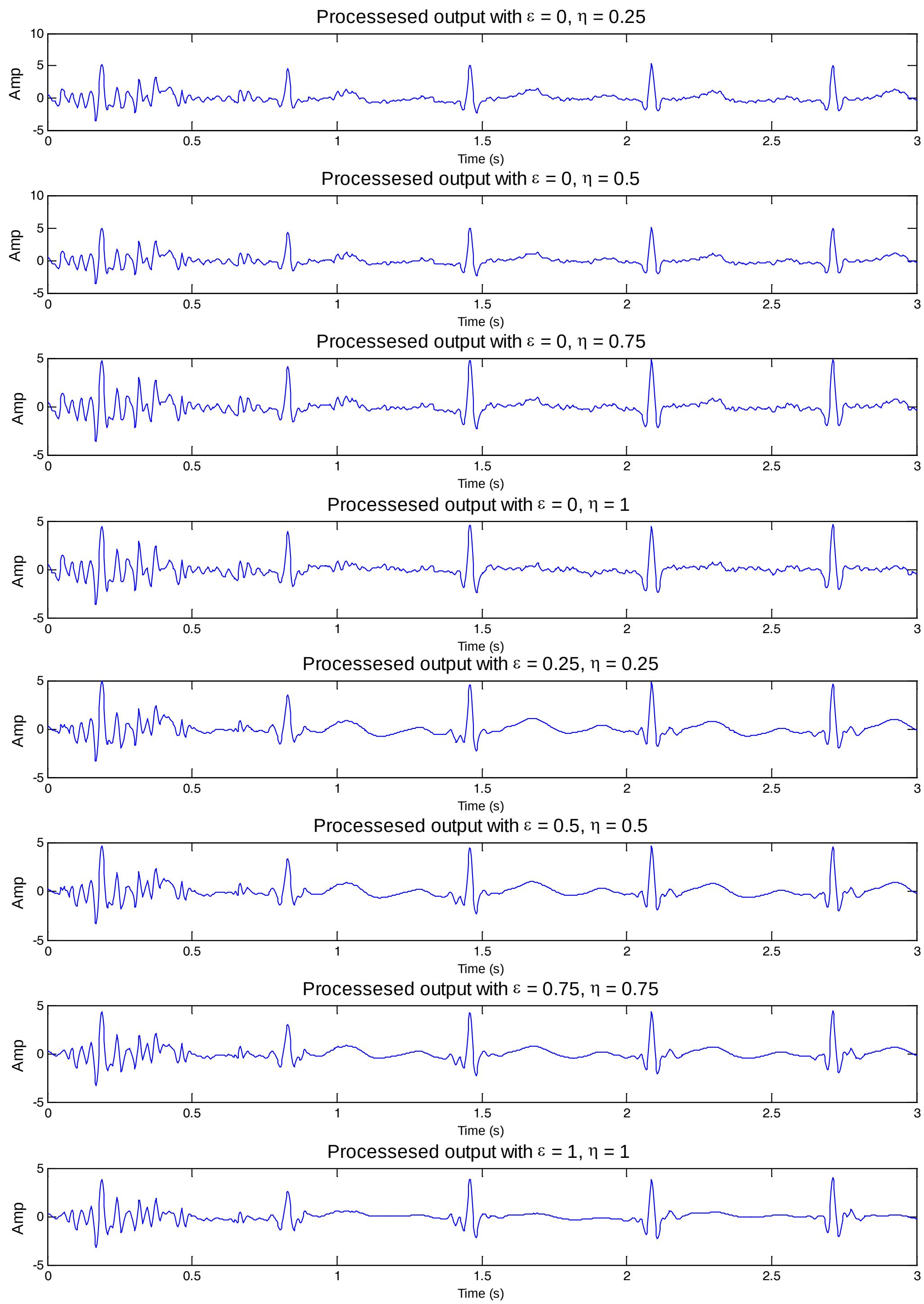

Figure B.8: 3-s ECG segment from record-16483 of "nsrdb" database corrupted by EMG noise at SNR = − 12 dB, and its denoising with various combination of ε and η.

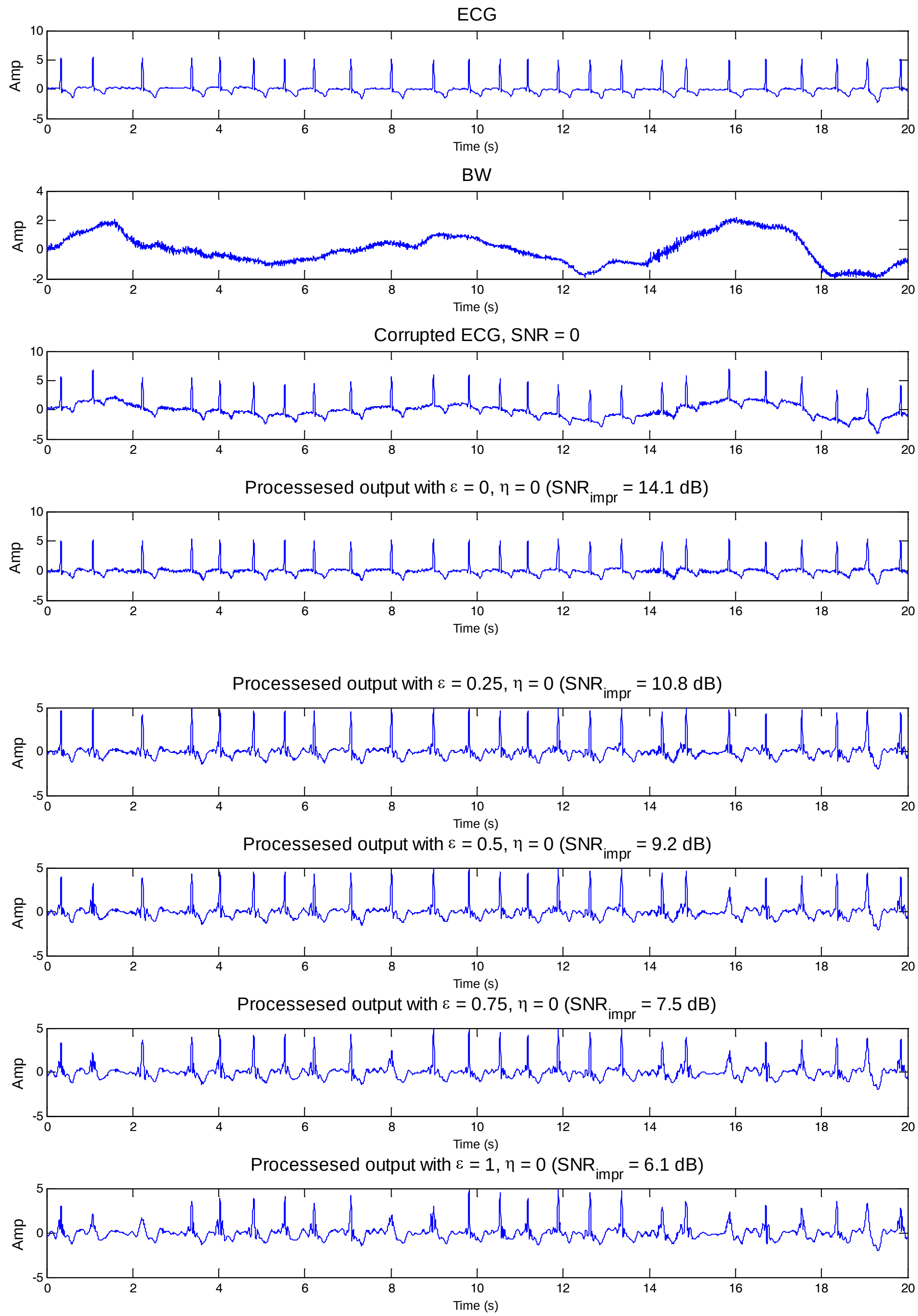

Figure B.9: Continued

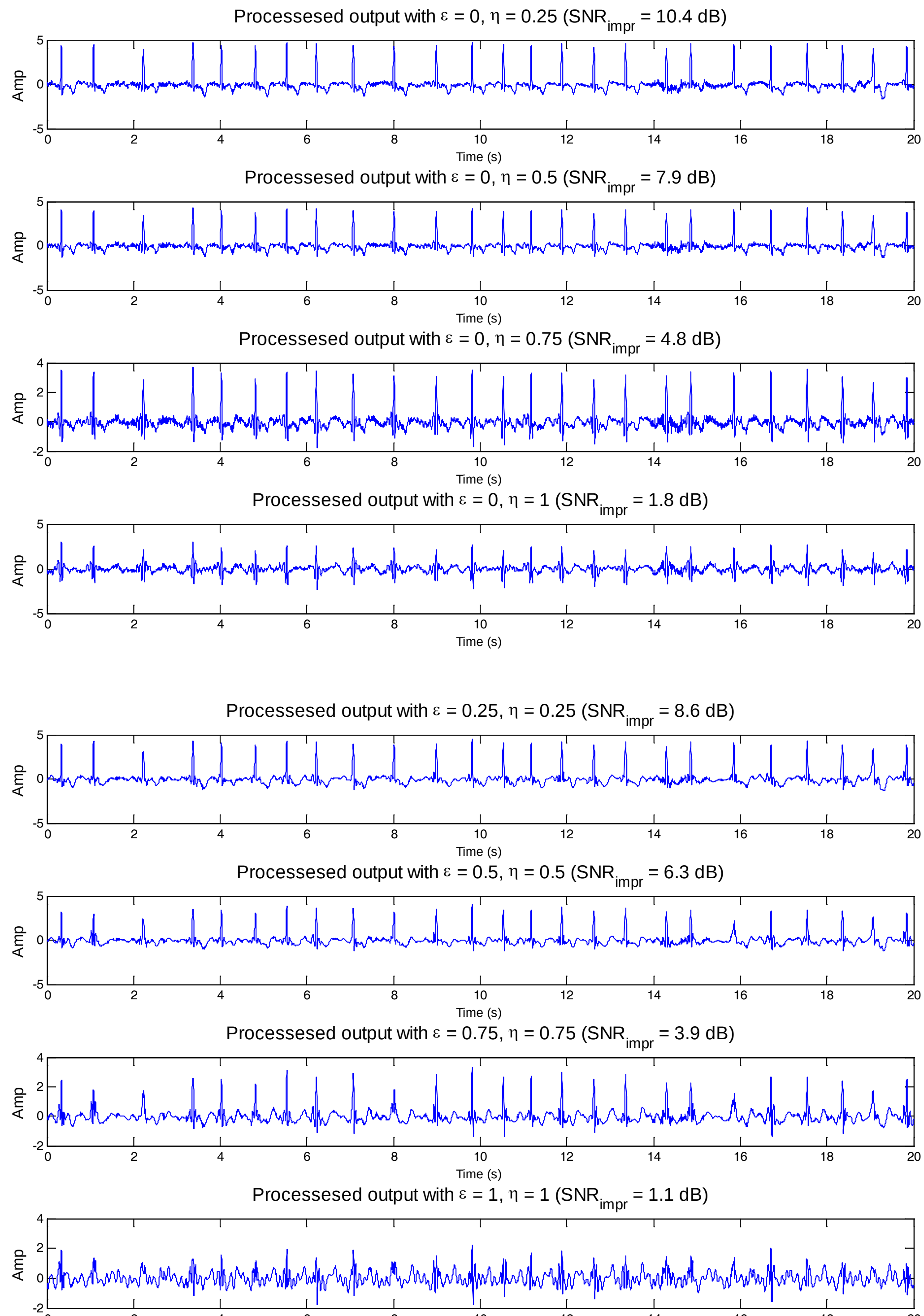

Figure B.9: 20-s ECG segment from record-219 of "mitdb" database corrupted by BW at SNR = 0 dB, and its denoising with various combination of ε and η.

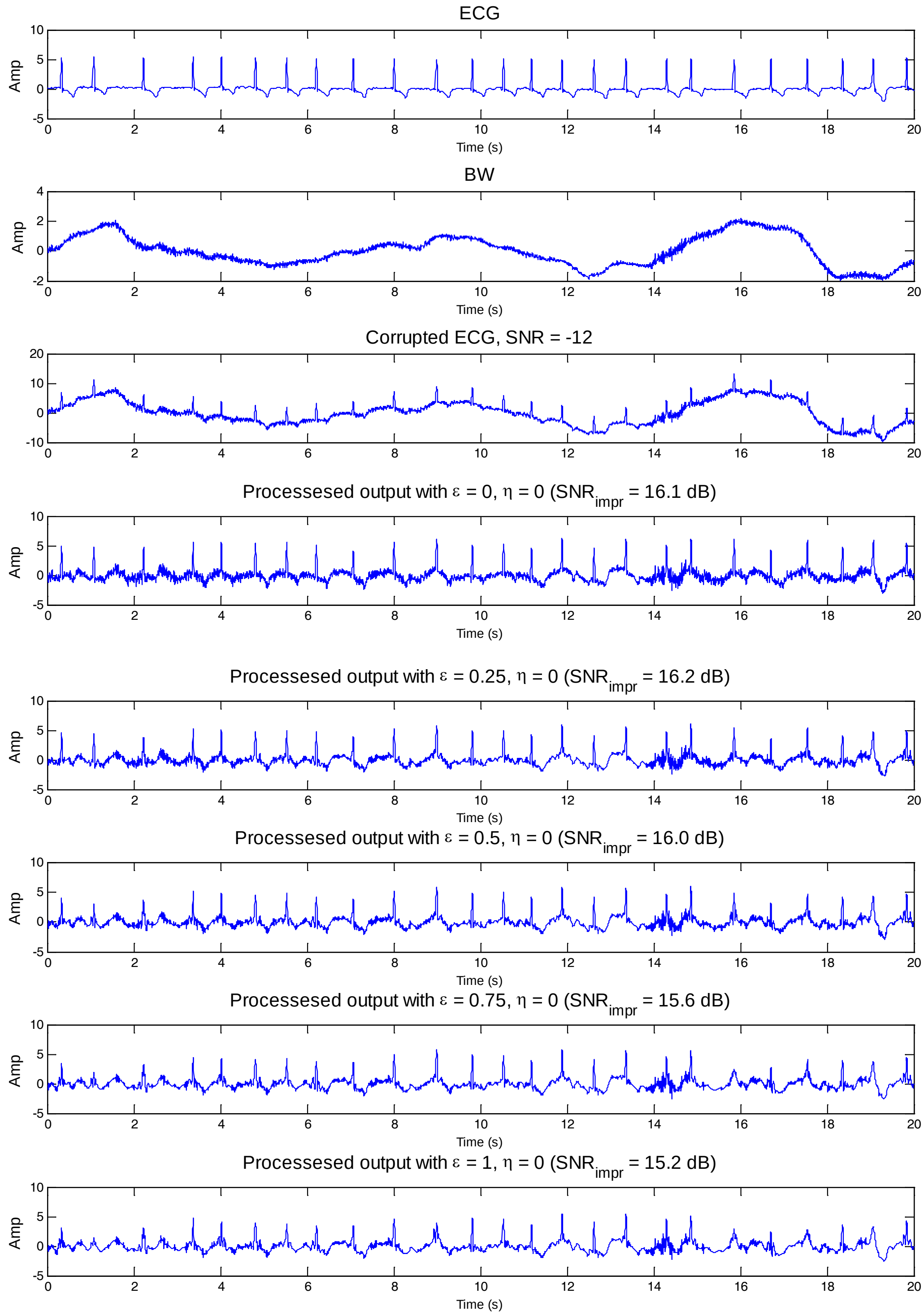

Figure B.10: Continued

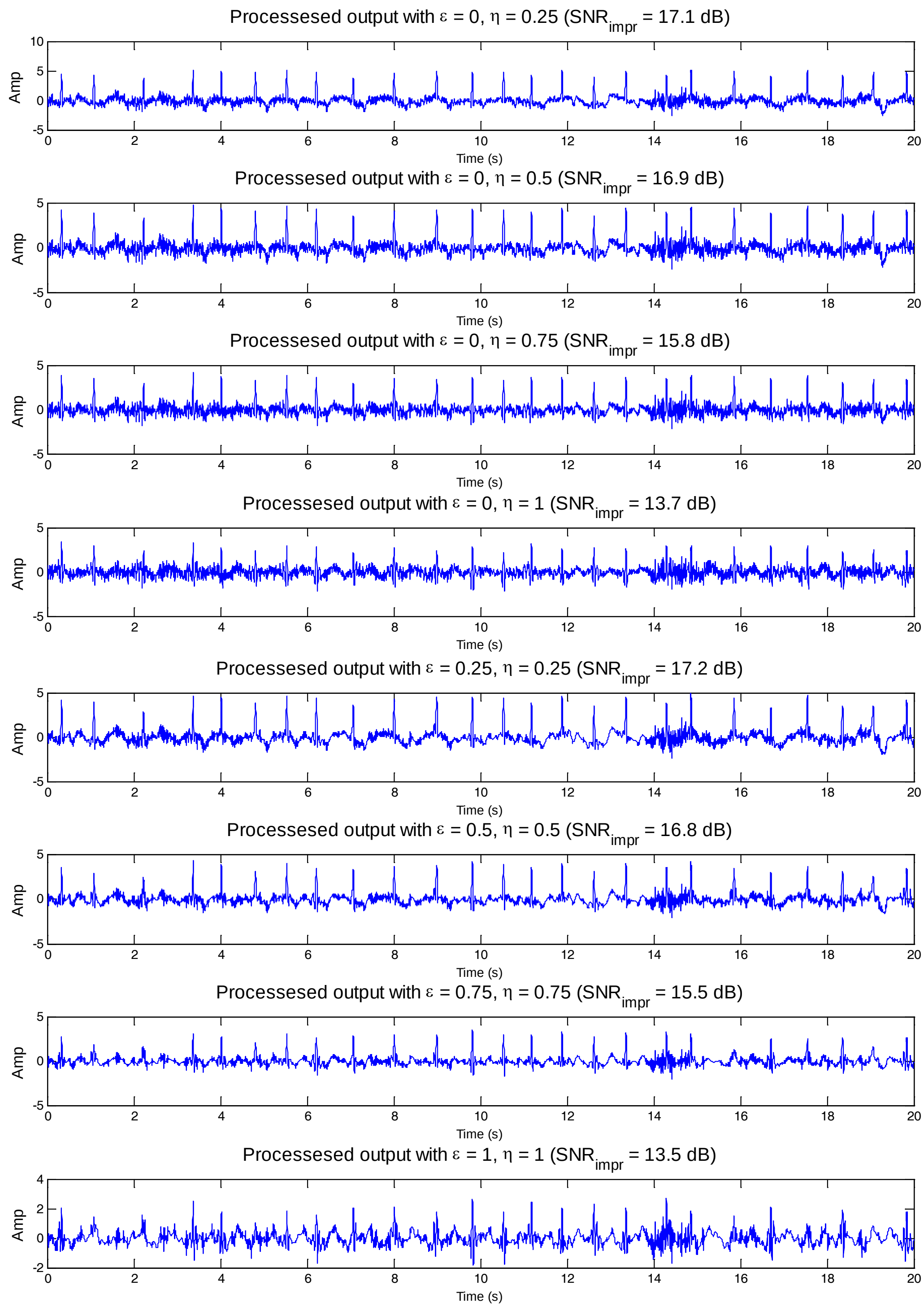

Figure B.10: 20-s ECG segment from record-219 of "mitdb" database corrupted by BW at SNR = − 12 dB, and its denoising with various combination of ε and η.

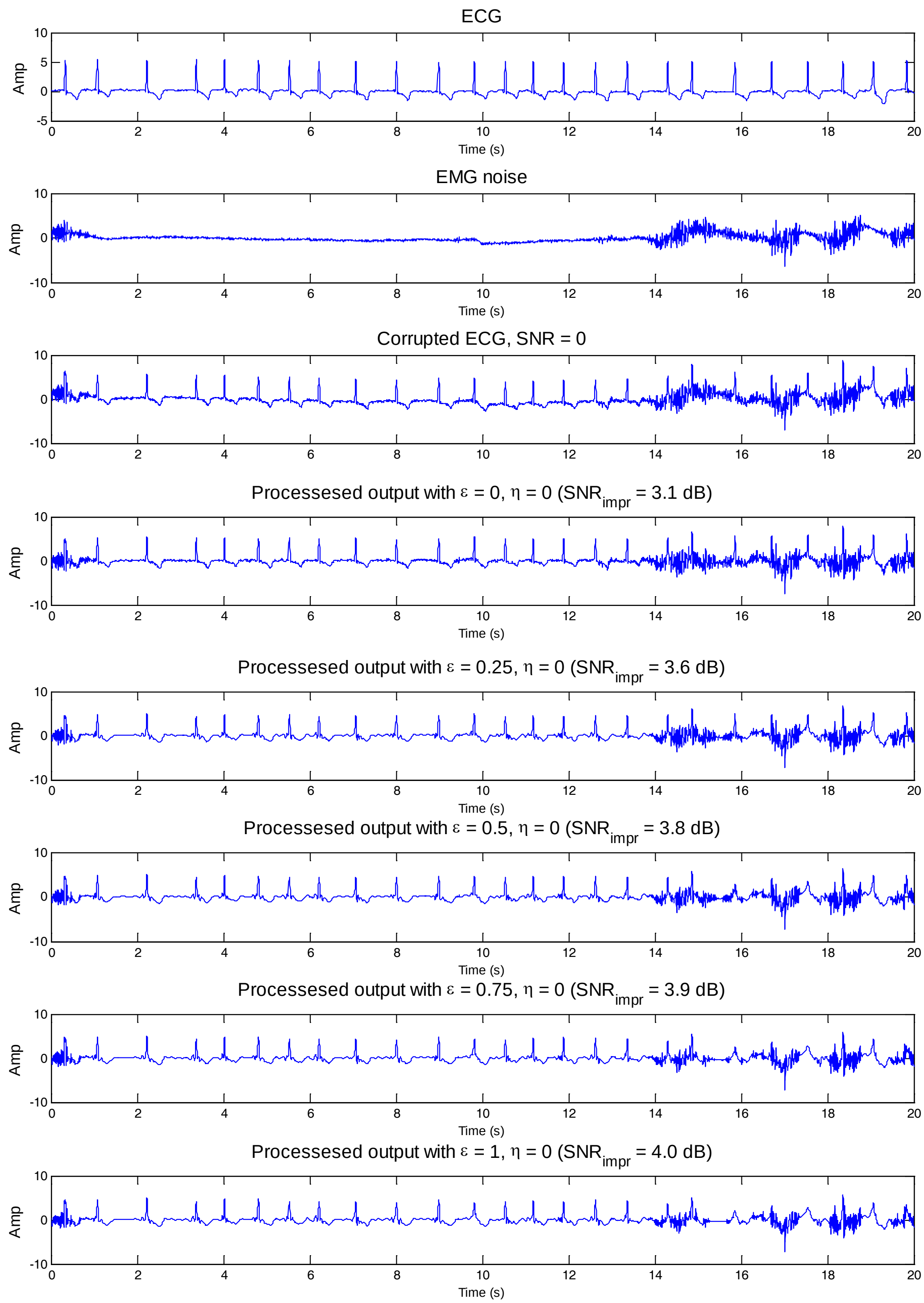

Figure B.11: Continued

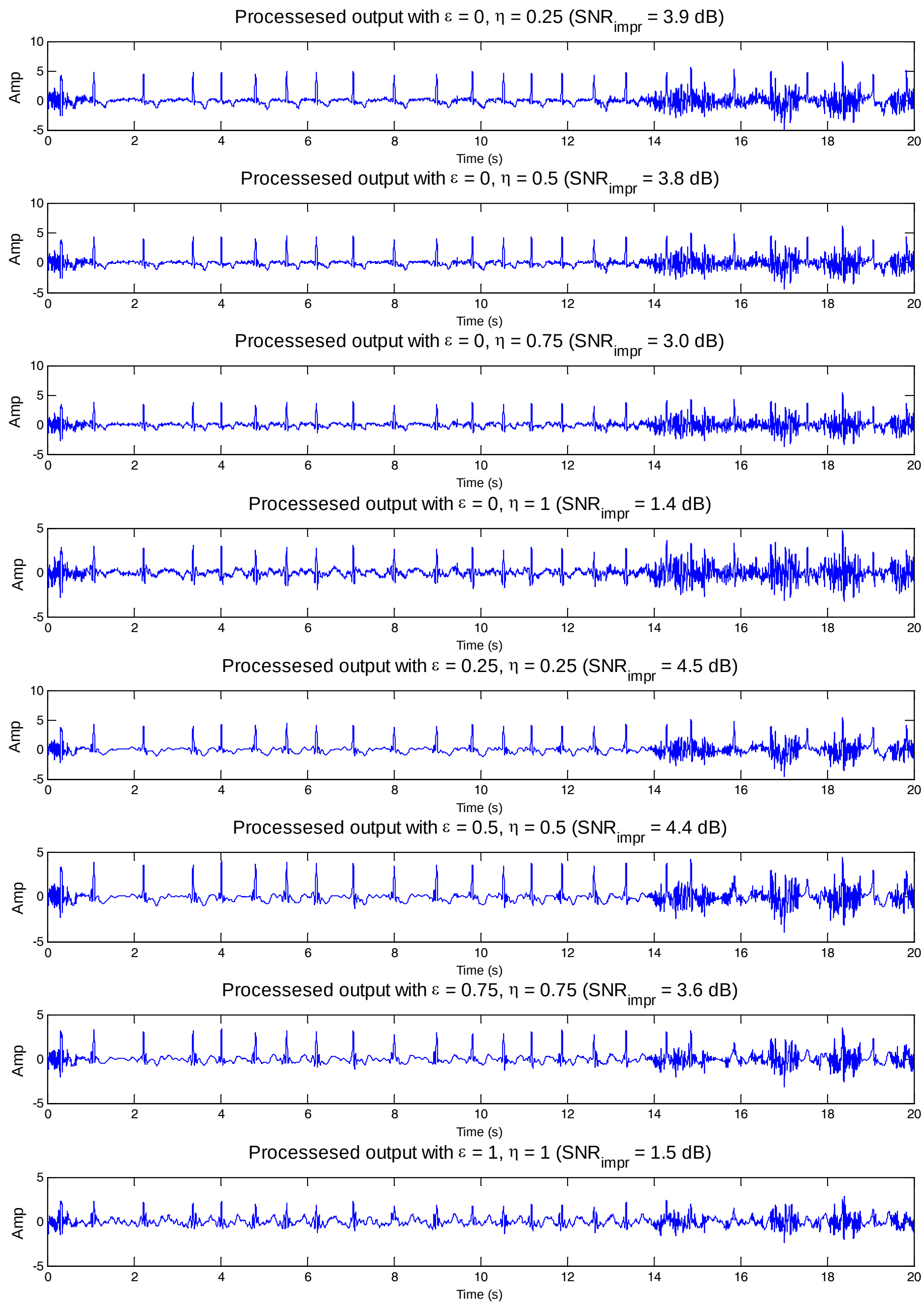

Figure B.11: 20-s ECG segment from record-219 of "mitdb" database corrupted by EMG noise at SNR = 0 dB, and its denoising with various combination of ε and η.

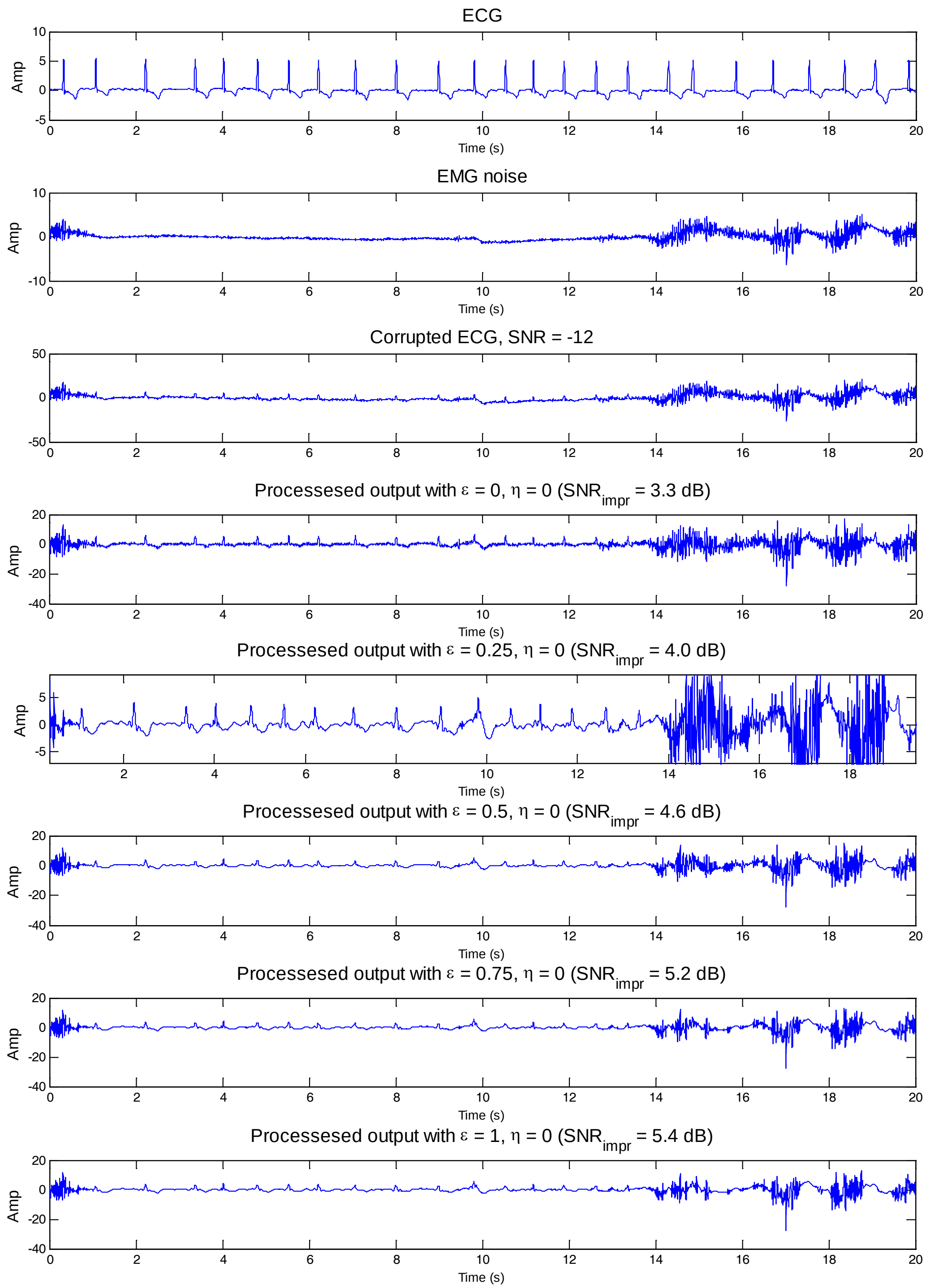

Figure B.12: Continued

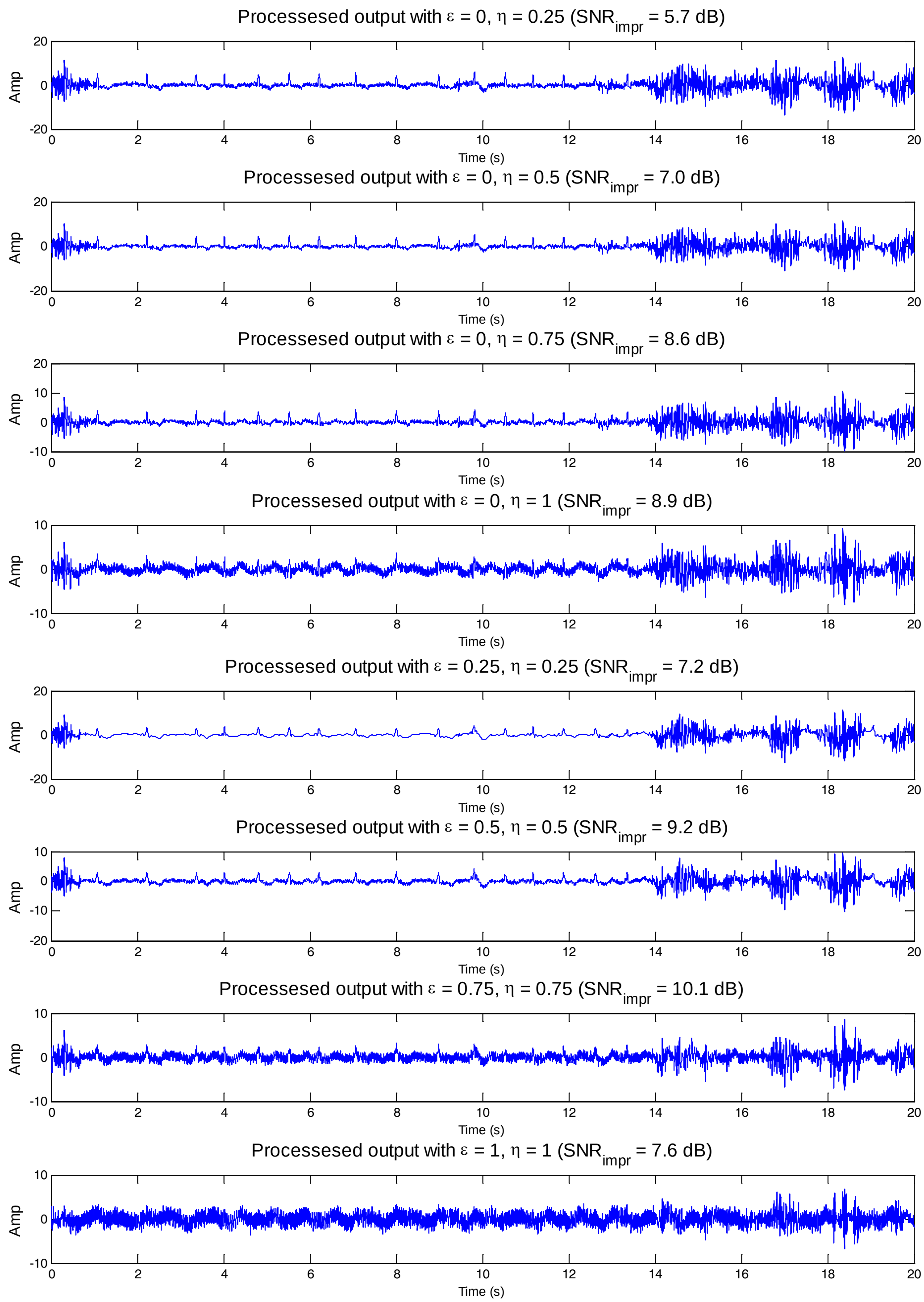

Figure B.12: 20-s ECG segment from record-219 of "mitdb" database corrupted by EMG noise at SNR = − 12 dB, and its denoising with various combination of ε and η.

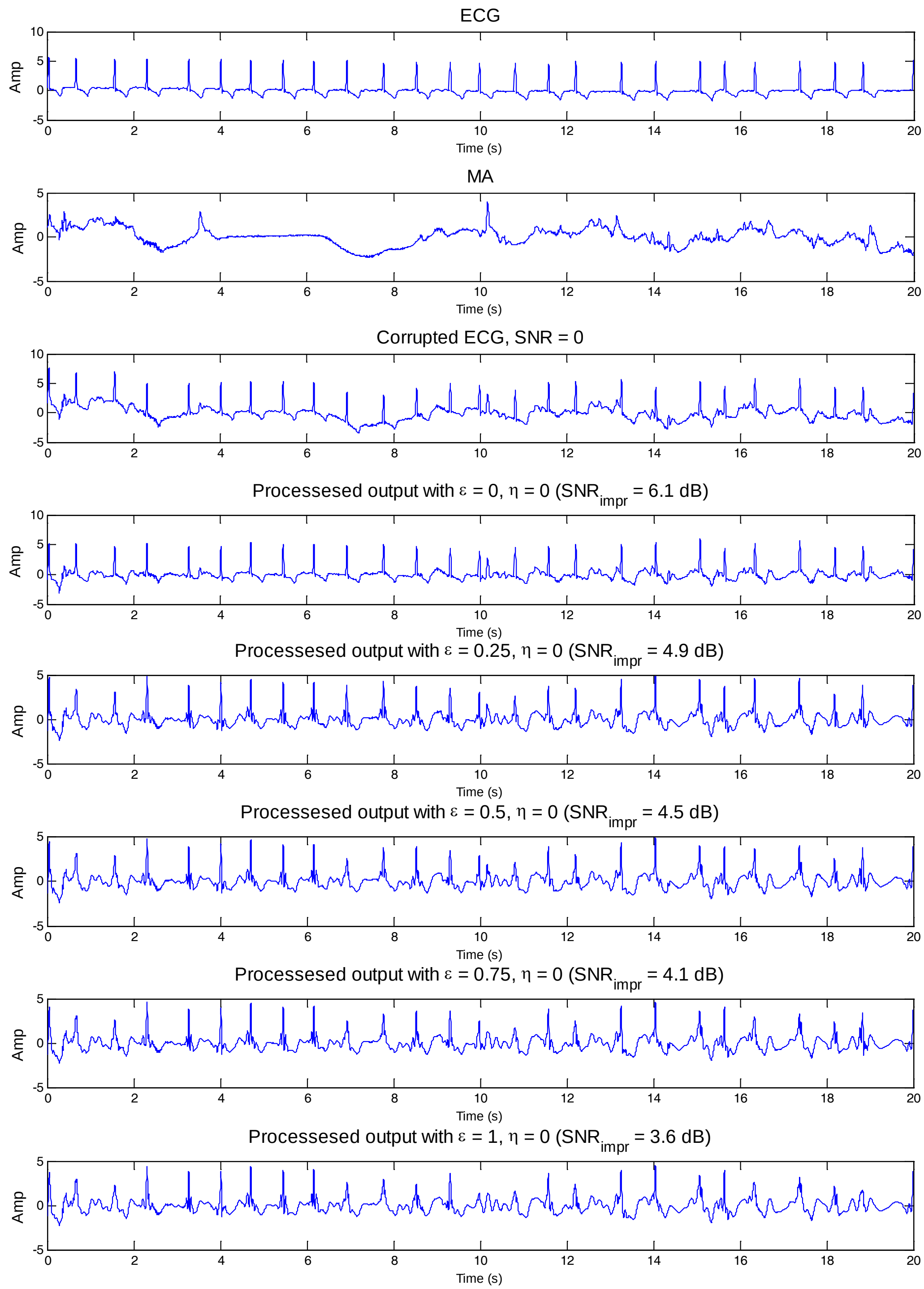

Figure B.13: Continued

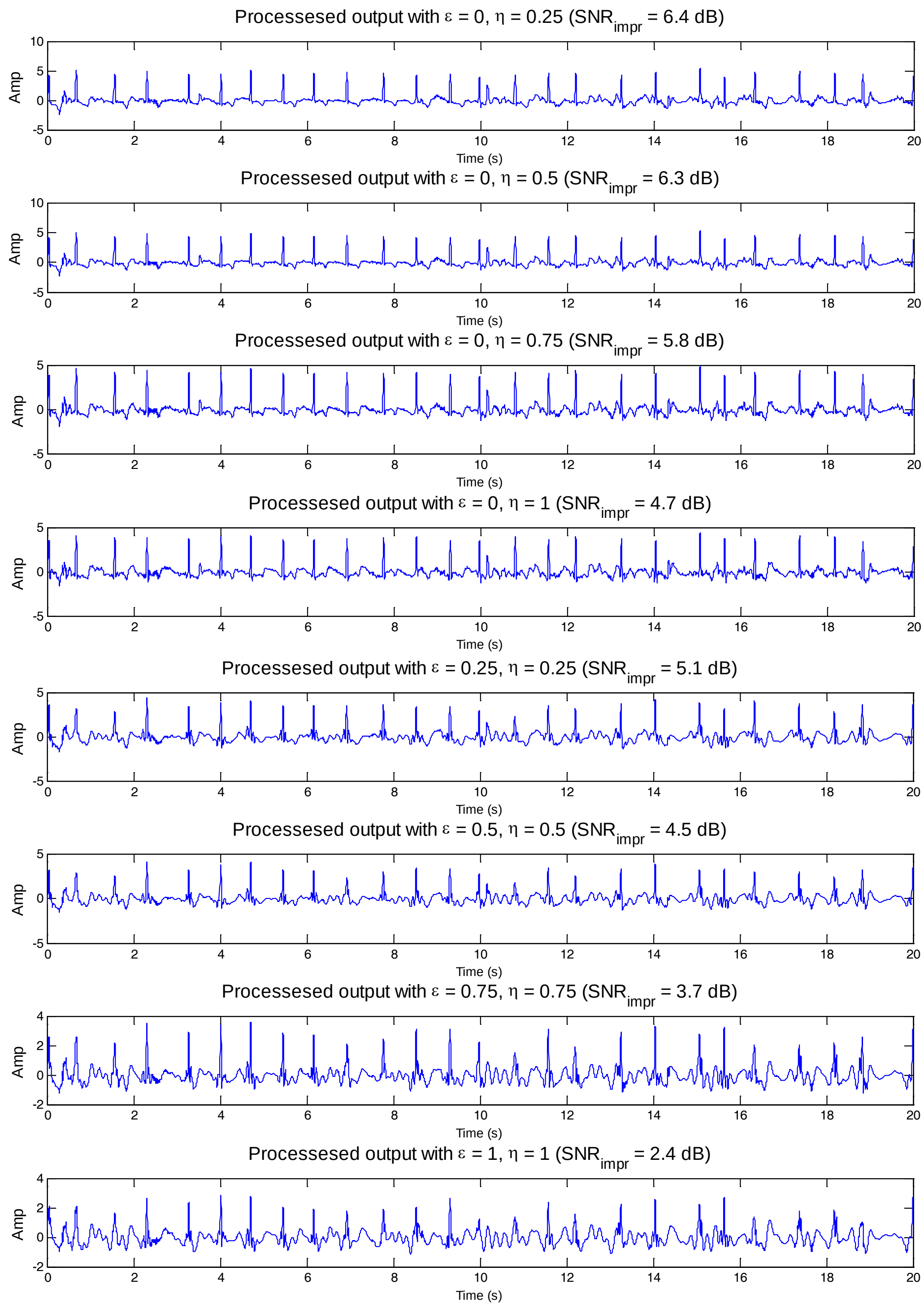

Figure B.13: 20-s ECG segment from record-219 of "mitdb" database corrupted by MA at SNR = 0 dB, and its denoising with various combination of ε and η.

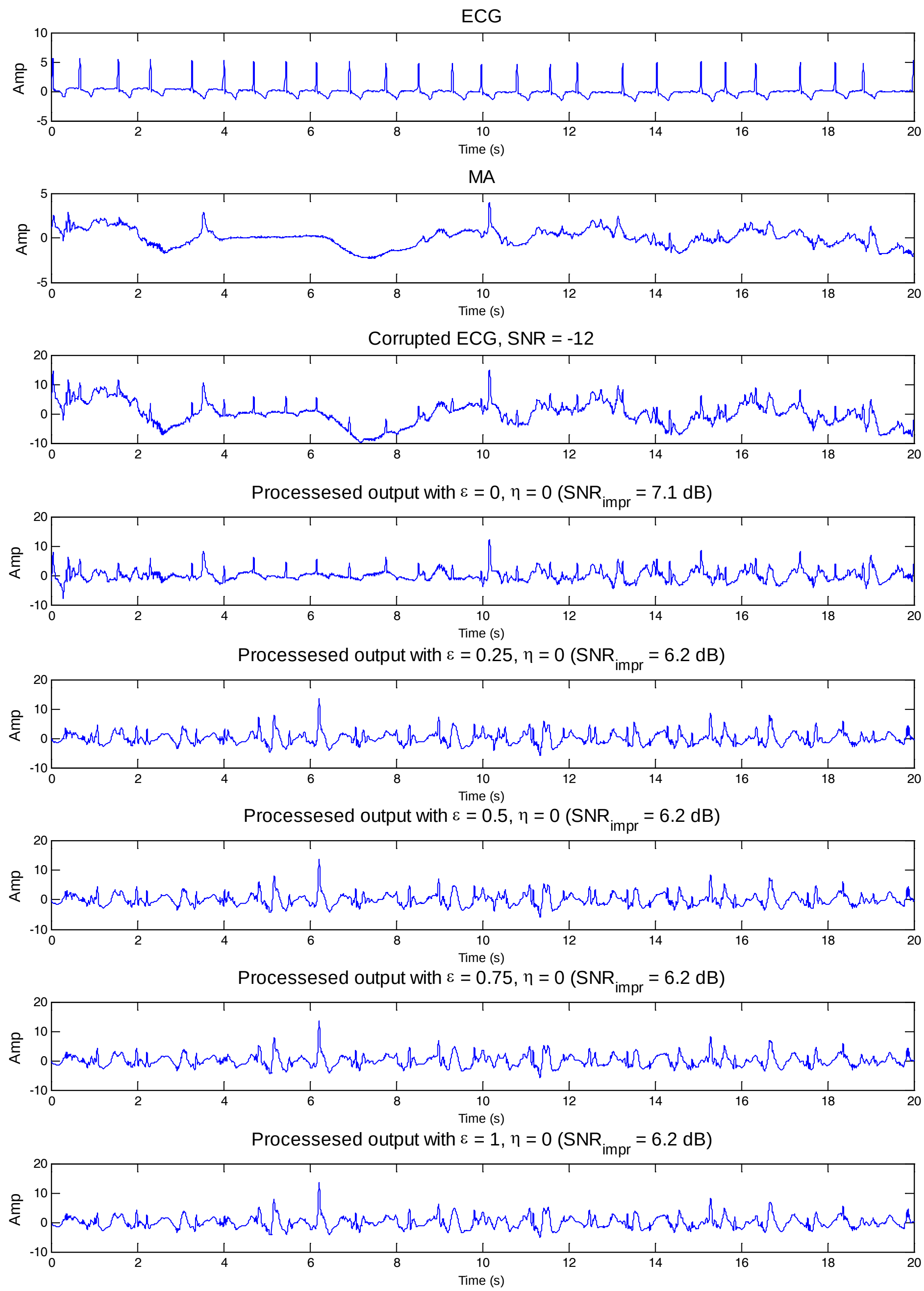

Figure B.14: Continued

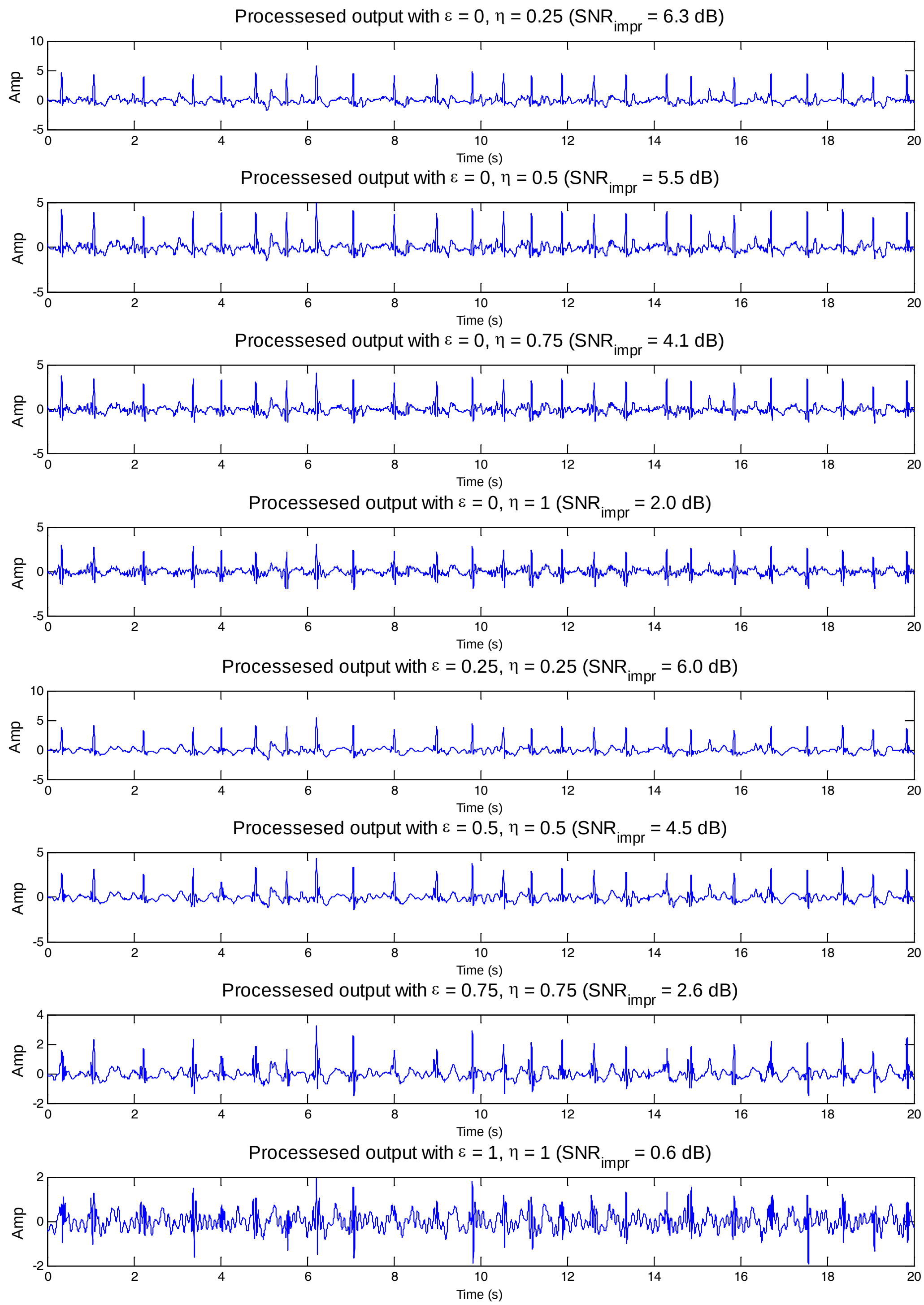

Figure B.14: 20-s ECG segment from record-219 of "mitdb" database corrupted by MA at SNR = − 12 dB, and its denoising with various combination of ε and η.

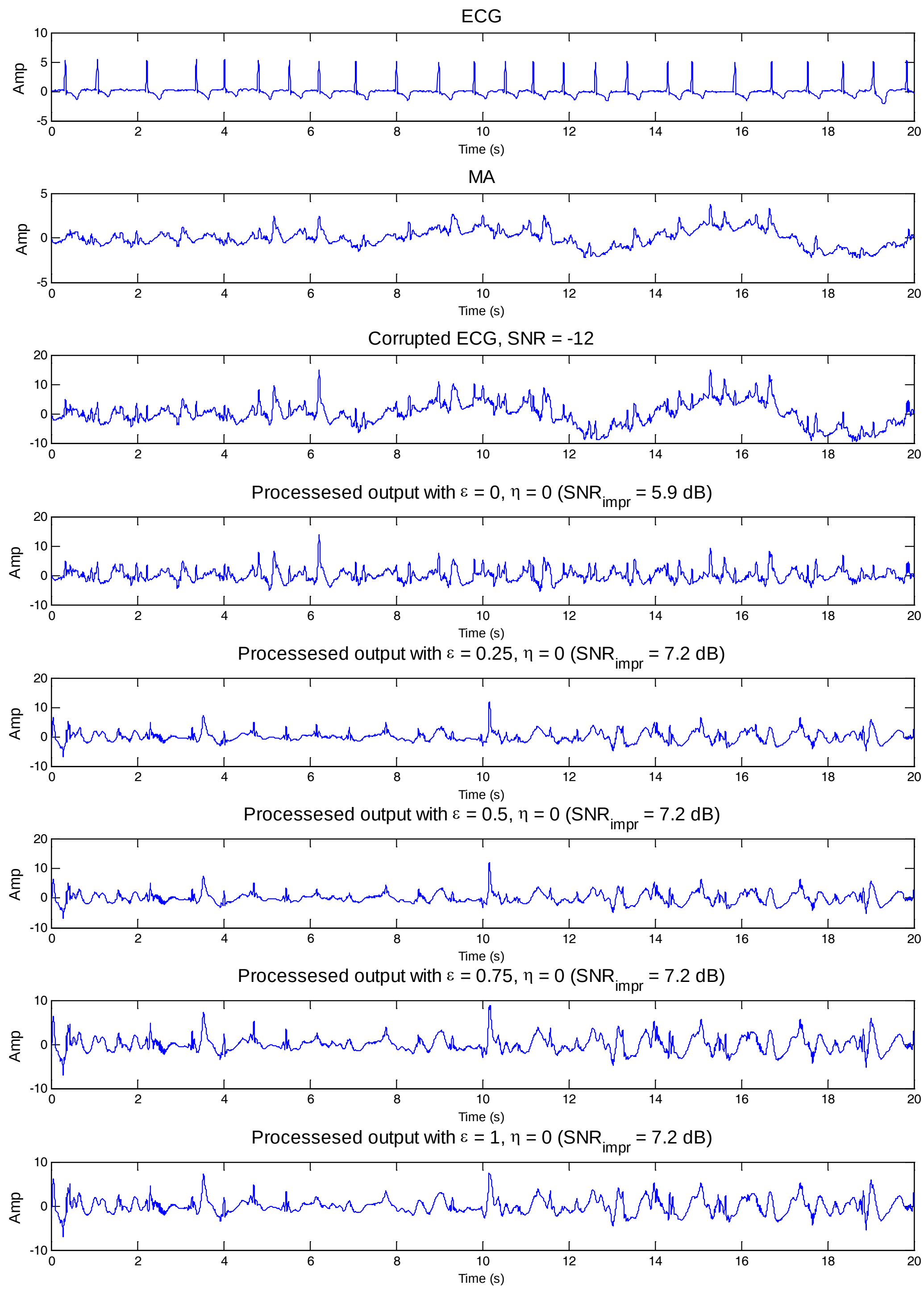

Figure B.15: Continued

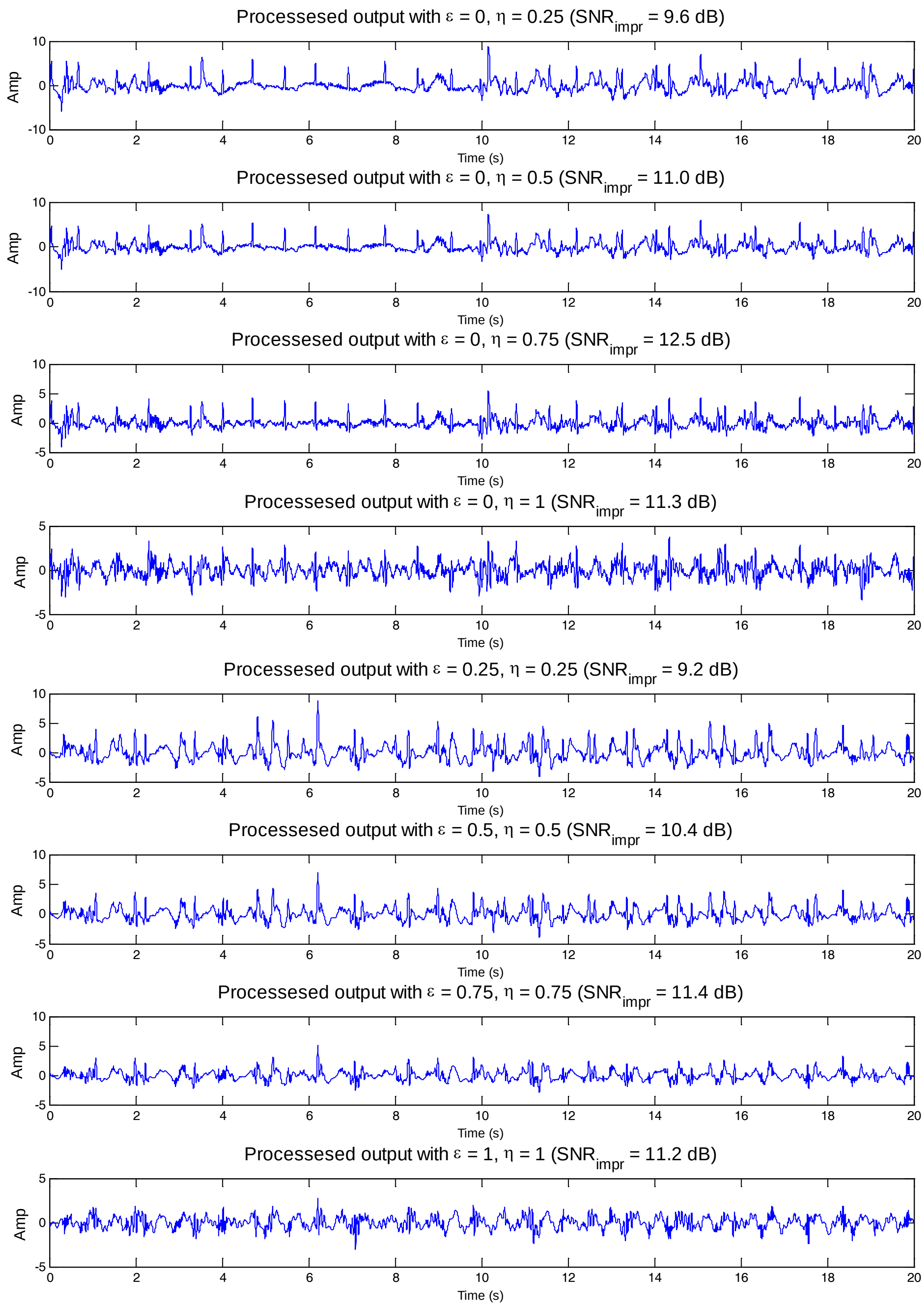

Figure B.15: 20-s ECG segment from record-219 of "mitdb" database corrupted by typical MA at SNR = − 12 dB, and its denoising with various combination of ε and η.

Appendix C

COMMONLY USED WAVELET AND SCALING FUNCTIONS

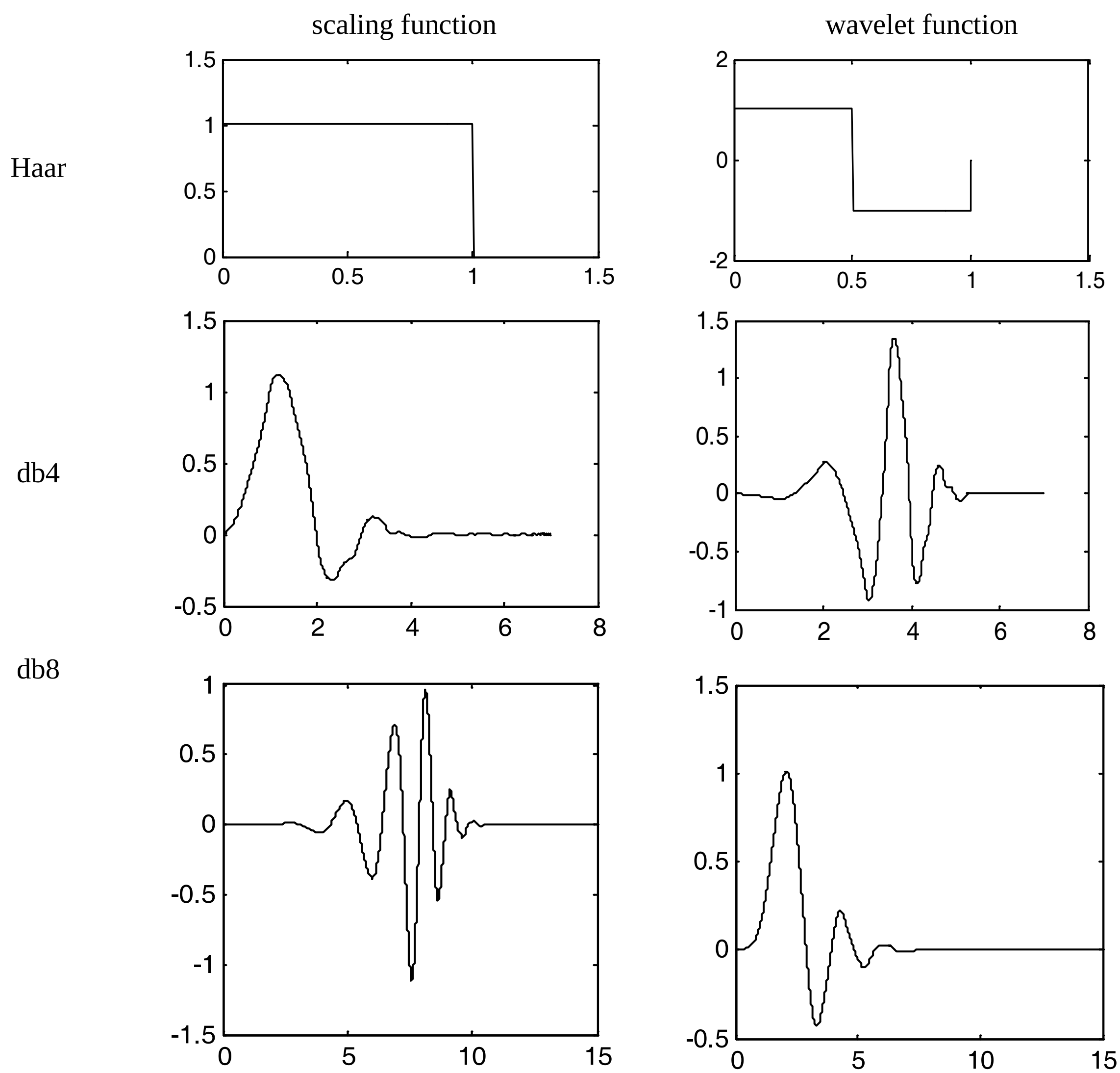

Figure C.1: Continued

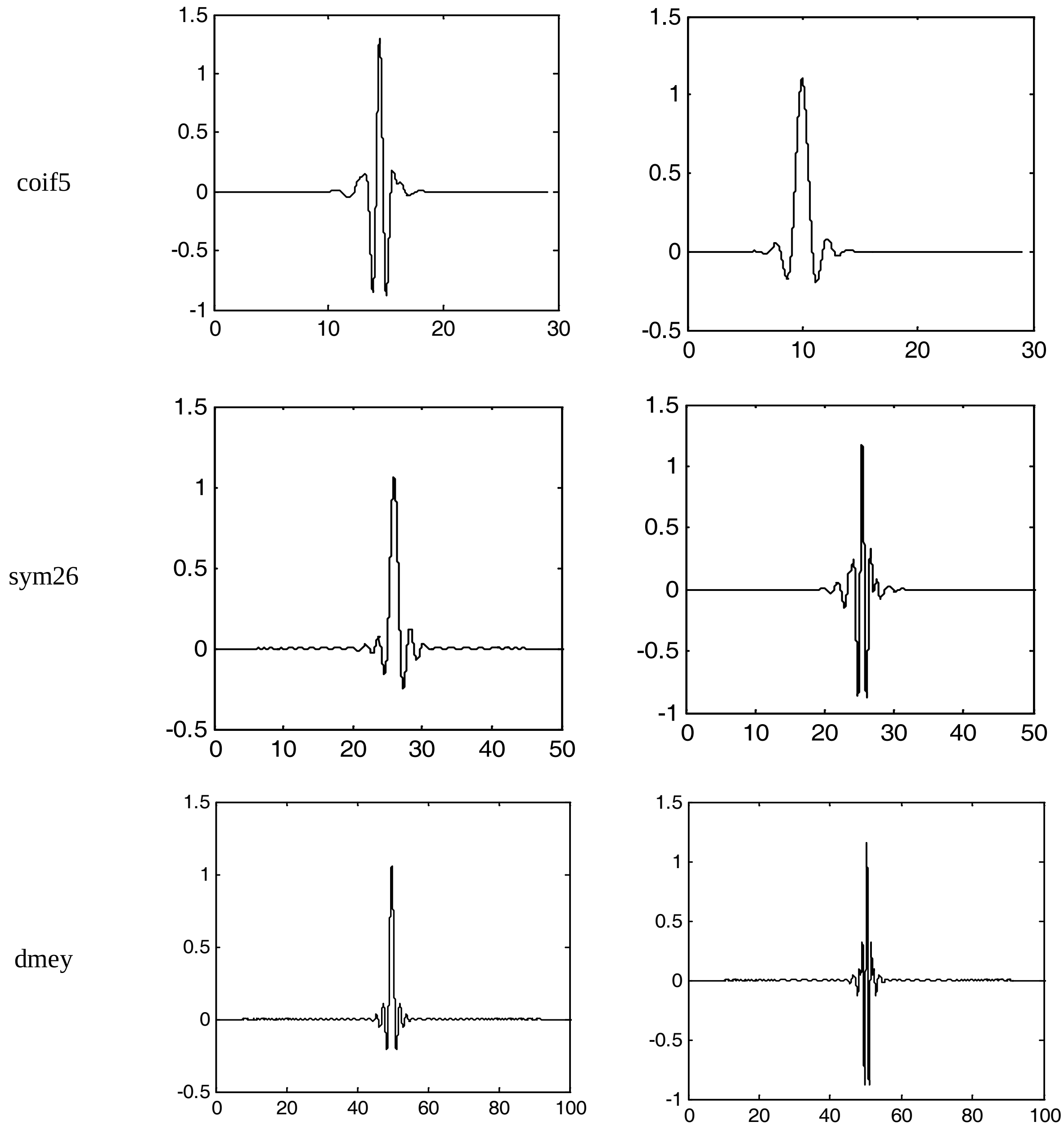

Figure C.1: Different wavelet and corresponding scaling function.

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Acknowledgement

I am grateful to Prof. P. C. Pandey for giving me an opportunity to work under his esteemed guidance for my M. Tech Project. I would like to express my deep gratitude for his valuable suggestions and constant encouragement throughout the project.

I thank P. Mithun and T. Sebastian for their support in the project. I would also like to express my sincere thanks to my lab mates Nitya and Santosh Waddi for their help and support during the project work. I am thankful to many friends especially Shravan and Karan for making life at IIT wonderful.

I am indebted to my parents, family members, and Lord Krishna for all types of support during my studies.

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