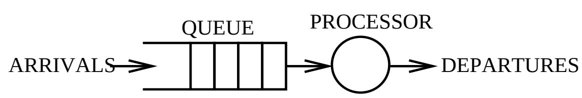
Department of Electrical Engineering, IIT Bombay

EE309 Computer Organization, Architecture and Microprocessors: Tutorial Sheet VII

Performance Evaluation and Stochastic Modeling of Computer Systems

Consider a M/M/1 modeling of a Computer System - Poisson Zeroth Order Markovian Arrival Process, Exponential Service Time Distribution, and a Uniprocessor System. The processor handles jobs in a First-In-First-Out (FIFO) manner. Assume the arrival rate and processor service rate to be λ and μ jobs per unit time, respectively.



- *The Arrival Process follows a Poisson Distribution, with the Zeroth-Order Markovian Assumption*
  $P_n(t)$ denotes the probability of n jobs arriving during time t. This follows a Poisson Distribution:

  $$P_n(t) \ \stackrel{\triangle}{=} \ \frac{(\lambda t)^n \ e^{-\lambda t}}{n!} \tag{1}$$

  The Zeroth-Order Markovian assumption implies that there is no dependence of a job arrival on the arrival of any other job, and that at most one job can arrival in an infinitesimal interval of time δt.

- *The Service Time follows an Exponential Distribution*
  $S(t)$ denotes the probability that a job is *not* serviced in time t:

  $$S(t) \ \stackrel{\triangle}{=} \ e^{-\mu t} \tag{2}$$

- *Traffic Intensity*
  We define the *Traffic Intensity* ρ as: $\rho \ \stackrel{\triangle}{=} \ \lambda/\mu$. We assume that $0 \leq \rho \leq 1$. In other words, jobs do not arrive at a rate faster than they can be serviced. Incidentally, a steady-state solution cannot exist for $\rho \geq 1$, since the arrival process would have saturated the processor and the queue would be growing without bounds.

- Use the term $p_n(t)$ to denote the probability that the system has n jobs in the queue at time t.

Problems:

1. Compute the steady state probability of the system having n jobs in the queue (p_n), as a function of ρ.

   Hint: Consider an infinitesimally small interval of δt. Find out $p_n(t+\delta t)$. Now, compute $\frac{d}{dt}\{p_n(t)\}$ from the above expression, as a recurrence relation involving $p_{n-1}(t)$, $p_n(t)$ and $p_{n+1}(t)$. Under steady state conditions, the above derivative is zero.

2. Find out the steady state probability that the processor is free (*i.e.*, has no job to process). This is also the fraction of the time the process is free to handle low priority I/O devices, or run maintenance programs.

   Hint: The answer is p_0.

3. Calculate the probability that more than k buffers will be required, as a function of the traffic intensity. This is a measure of the length of the queue to ensure that it does not overflow frequently.

4. Compute the average queue length $\overline{Q}$.

5. What is the average waiting time (*i.e.*, the Average response time) ? You may use Little's Formula relating the average queue length ($\overline{Q}$) to the average waiting time ($\overline{W}$): $\overline{Q} = \lambda\, \overline{W}$.