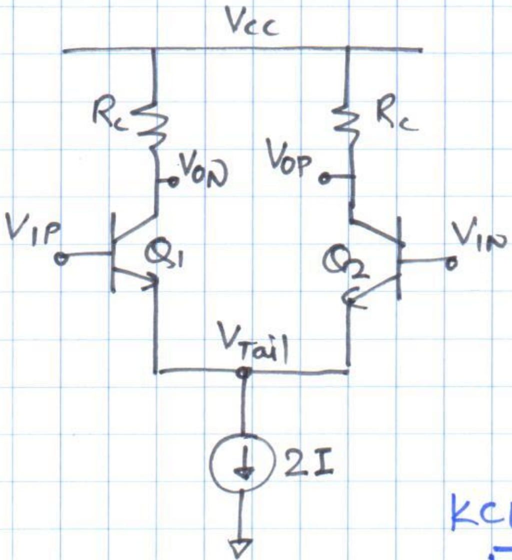
Large signal Analysis (Bipolar Diff pair)

ignore r_o & base current



$$V_{IP} - V_{BE_1} = V_{Tail} = V_{IN} - V_{BE_2}$$

$$V_{IP} - V_{IN} = V_{BE_1} - V_{BE_2}$$

$$= V_T \ln \frac{I_{C_1}}{I_{S_1}} - V_T \ln \frac{I_{C_2}}{I_{S_2}}$$

$$\boxed{V_{IP} - V_{IN} = V_T \ln\left(\frac{I_{C_1}}{I_{C_2}}\right)} \quad \text{— (1)}$$

KCL @ V_{TAIL}.

$$\boxed{I_{C_1} + I_{C_2} = 2I} \quad \text{— (2)}$$

from (1) $I_{C_1} = I_{C_2} \cdot \exp\left(\frac{V_{IP} - V_{IN}}{V_T}\right)$

Substitute in (2)

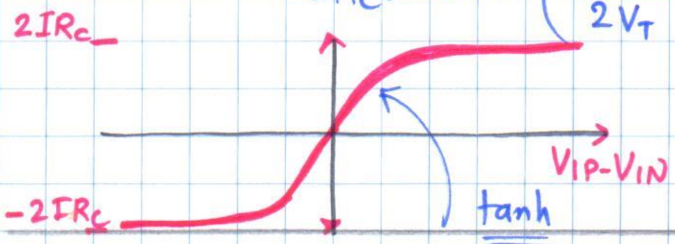
$$I_{C_2} \exp\left(\frac{V_{IP} - V_{IN}}{V_T}\right) + I_{C_2} = 2I$$

$$I_{C_2} = \frac{2I}{1 + \exp\left(\frac{V_{IP} - V_{IN}}{V_T}\right)} \quad \text{— (3)}$$

$$I_{C_1} = \frac{2I \cdot \exp\left(\frac{V_{IP} - V_{IN}}{V_T}\right)}{1 + \exp\left(\frac{V_{IP} - V_{IN}}{V_T}\right)}$$

$$\underline{V_{OP} - V_{ON}} = -R_C\left(I_{C_2} - I_{C_1}\right) = R_C(2I)\,\frac{\exp\left(\frac{V_{IP} - V_{IN}}{V_T}\right) - 1}{\exp\left(\frac{V_{IP} - V_{IN}}{V_T}\right) + 1}$$

$$= 2R_C I \cdot \tanh\left(\frac{\overbrace{V_{IP} - V_{IN}}^{V_{DM}}}{2V_T}\right) \begin{cases} \to -2IR_C & (V_{IP} - V_{IN}) \ll 0 \\ \to +2IR_C & (V_{IP} - V_{IN}) \gg 0 \end{cases}$$

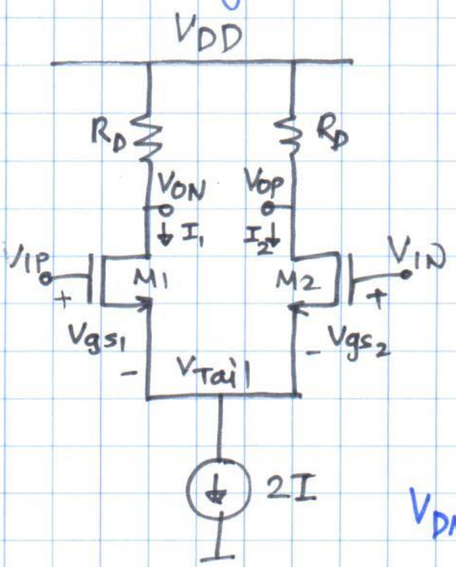


Note: No inversion since labling of Vop & VON already is inverted.

Large Signal Analysis (MOSFET Diff. pair)

ignore r_o & Body effect

M_1 & M_2 are matched.



$$K_n = \mu_n C_{ox}\left(\frac{W}{L}\right)$$

$$I_1 = \frac{K_n}{2}(V_{gs_1} - V_{TH})^2 \rightarrow V_{gs_1} = V_{TH} + \sqrt{\frac{2I_1}{K_n}}$$

$$I_2 = \frac{K_n}{2}(V_{gs_2} - V_{TH})^2 \rightarrow V_{gs_2} = V_{TH} + \sqrt{\frac{2I_2}{K_n}}$$

$$V_{DM} = V_{1P} - V_{1N} = V_{gs_1} - V_{gs_2} = \sqrt{\frac{2I_1}{K_n}} - \sqrt{\frac{2I_2}{K_n}}$$

also
$$\left.\begin{aligned} I_1 + I_2 &= 2I \\ I_1 - I_2 &= \Delta I \end{aligned}\right\} \Rightarrow \begin{aligned} I_1 &= I + \frac{\Delta I}{2} \\ I_2 &= I - \frac{\Delta I}{2} \end{aligned}$$

$$V_{DM}^2 = \frac{2I_1}{K_n} + \frac{2I_2}{K_n} - \frac{4}{K_n}\sqrt{I_1 I_2}$$

$$V_{DM}^2 = \frac{2}{K_n}(I_1 + I_2) - \frac{4}{K_n}\sqrt{\left(I + \frac{\Delta I}{2}\right)\left(I - \frac{\Delta I}{2}\right)}$$

$$\frac{K_n}{4}V_{DM}^2 = I - \sqrt{I^2 - \left(\frac{\Delta I}{2}\right)^2}$$

Rearrange

$$\sqrt{I^2 - \left(\frac{\Delta I}{2}\right)^2} = I - \frac{K_n}{4}V_{DM}^2 \qquad \text{Square both sides}$$

$$I^2 - \left(\frac{\Delta I}{2}\right)^2 = I^2 - \frac{K_n}{2} I V_{DM}^2 + \frac{K_n^2}{16}V_{DM}^4$$

$$\frac{\Delta I^2}{4} = \frac{K_n}{2} I V_{DM}^2 - \frac{K_n^2}{16}V_{DM}^4$$

$$\Delta I^2 = 2K_n I V_{DM}^2 - \frac{K_n^2}{4}V_{DM}^4$$

$$\Delta I^2 = \frac{K_n^2}{4}V_{DM}^2\left[\frac{2K_n I \cdot 4}{K_n^2} - V_{DM}^2\right]$$

$$\Delta I = \frac{K_n}{2}V_{DM}\sqrt{\frac{8I}{K_n} - V_{DM}^2}$$

valid
$$-\sqrt{\frac{8I}{K_n}} \leq V_{DM} \leq \sqrt{\frac{8I}{K_n}}$$

When current completely switches to one side → $V_{DM} = V_{Max}$.

$V_{DM} = V_{max}$ $\quad\quad \Delta I = 2I$

$$2I = \Delta I = \frac{k_n}{2} V_{max} \sqrt{\frac{8I}{k_n} - V_{max}^2} \longrightarrow \text{Solve} \rightarrow V_{max} = \sqrt{\frac{4I}{k_n}}$$

$$\Downarrow$$

$$V_{max}^2 = \frac{4I}{k_n}$$

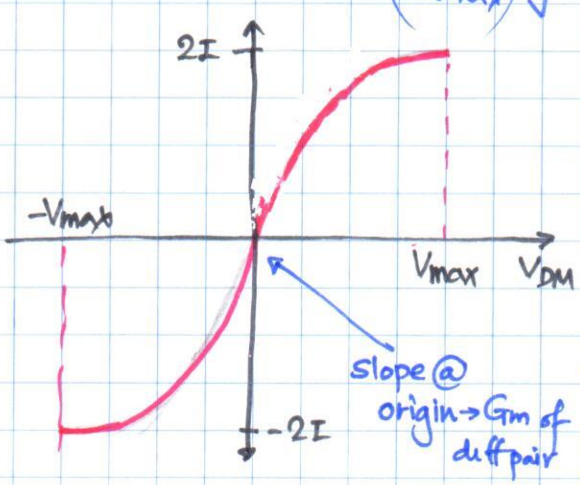
$$\Delta I = \frac{k_n}{2} V_{DM} \sqrt{\frac{8I}{k_n} - V_{DM}^2}$$

$$k_n = \frac{4I}{V_{max}^2} \quad \text{(sub.)}$$

$$= \frac{4I}{2 \cdot V_{max}^2} \cdot V_{DM} \cdot \sqrt{2V_{max}^2 - V_{DM}^2}$$

$$= \frac{2I}{V_{max}^2} \times V_{DM} \cdot \sqrt{2}\, V_{max} \sqrt{1 - \frac{1}{2}\left(\frac{V_{DM}}{V_{max}}\right)^2}$$

$$\Delta I = 2\sqrt{2}\, I \left(\frac{V_{DM}}{V_{max}}\right) \sqrt{1 - \frac{1}{2}\left(\frac{V_{DM}}{V_{max}}\right)^2}$$



* In bipolar case as $V_{DM} \uparrow$ ΔI approaches $2I$ asymptotically.
* In MOSFET case @ $V_{DM} = V_{max}$ $\Delta I = 2I$.
* Key point: One of the MOSFET turns off ($V_{GS} = V_{TH}$)

@ $V_{DM} = 0 \Rightarrow I_1 = I_2 = I = \frac{k_n}{2}(V_{GS} - V_{TH})^2 = \frac{k_n}{2} V_{Dsat}^2$

$$V_{Dsat} = \sqrt{\frac{2I}{k_n}} \Rightarrow V_{max} = \sqrt{\frac{4I}{k_n}} = \sqrt{2}\, V_{Dsat}$$

If $V_{Dsat} = 200mV \rightarrow V_{max} = 283mV$ ← current switched completely

$$\Delta I = 2\sqrt{2}\, I \left(\frac{V_{DM}}{V_{max}}\right)\sqrt{1 - \frac{1}{2}\left(\frac{V_{DM}}{V_{max}}\right)^2}$$

for $\frac{V_{DM}}{V_{max}} \ll 1$

$$\Delta I \simeq 2\sqrt{2}\, I \left(\frac{V_{DM}}{V_{max}}\right)$$

$$\frac{\Delta I}{V_{DM}} \cong \frac{2\sqrt{2}\, I}{V_{max}} \cong \frac{2\sqrt{2}\, I}{\sqrt{4I}} \cdot \sqrt{k_n} = \sqrt{2}\sqrt{I}\sqrt{k_n}$$

$$= \sqrt{2 k_n I} = \underbrace{\sqrt{2 \mu_n C_{ox} \frac{W}{L} I}}$$

G_m of diff pair

g_m of Each transistor